MATHEMATICS Paper–I

(Solid Geometry)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt five questions, selecting at least two questions from each Section.

SECTION-I

1 (*a*) Shift the origin to a suitable point so that the equation.

$$2x^2 + 3y^2 + z^2 + xy + zx - x - 10y - 4z + 22 = 0.$$

is transformed into an equation in which the first degree terms are absent.

(*b*) If $< L_1, m_1, n_1 >, < L_2, m_2, n_2 >$, be the direction cosines of two lines inclined at an angle θ , show that the directioncosines of the directions bisecting them are :

$$< \left(\frac{L_1 + L_2}{2}\right)\sec\frac{\theta}{2}, \left(\frac{m_1 + m_2}{2}\right)\sec\frac{\theta}{2}, \left(\frac{n_1 + n_2}{2}\right)\sec\frac{\theta}{2} >$$ 2, 4

2. (*a*) Find the equation of the sphere circumscribing the tetrahedron whose faces are $x = 0,\ y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(*b*) Find the locus of the centres of the spheres passing through the fixed point (0,2,0) and touching the plane $y = 0$. 3, 3

3. (*a*) Prove that every sphere through the circle :
$x^2 + y^2 + 2ax + r^2 = 0,\ z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2,\ y = 0$

(*b*) Find the equation of the sphere which belongs to the coaxial system whose limiting points are (1,2,0), (2,2,0), and which passes through the point (3,-1,0). 3, 3

4. (*a*) Find the equation of the right circular cylinder described on the cirle through the points (2,0,0), (0,2,0), (0,0,2) as the guiding circle.

(*b*) Find the equation of the cyclinder whose generators are parallel to the line $\frac{x-4}{3} = \frac{y}{5} = \frac{z-3}{-4}$ and whose guiding curve is the hyperbola $4x^2 - 3y^2 = 5,\ z = 2$. 3, 3

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SECTION-II

5. (a) The section of a cone whose vertex is P and guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane x = 0 is a rectangular hyperbola. Show that locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

   (b) Find the equation of cone whose vertex is at (5,4,3) and guiding curve is $3x^2+2y^2=6,\ y+z=0$. 3, 3

6. (a) Show that the plane $6x+3y-2z=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines.

   (b) Prove that the tangent planes to the cone $Lyz+mzx+nxy=0$ are at right angles to the generators of the cone

   $L^2x^2+m^2y^2+n^2z^2-2mnyz-2nLzx-2Lmxy=0$ 3, 3

7. (a) Show that $33x^2+13y^2-95z^2-144yz-96zx-48xy=0$ represents a right circular cone whose axis is the line $3x=2y=z$. Find its vertical angle.

   (b) Show that the locus of the foot of the perpendicular from the centure of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ to any of its tangent plane is $(x^2+y^2+z^2)^2=a^2x^2+b^2y^2+c^2z^2$. 3,3

8. (a) Reduce the equation

   $11x^2+10y^2+6z^2-8yz+4zx-12xy+72x-72y+36z+150=0$ into the standard form and show that it represents an ellipsoid. Also find the equations of the axes.

   (b) If a right circular cone has three mutually perpendicular generators, then show that its vertical angle is $\tan^{-1}\sqrt{2}$ 4,2

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