MATHEMATICS Paper–I

(Plane Geometry)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section-A

1. (a) Transform $5x^2 - 2xy + 5y^2 + 2x - 10y - 7 = 0$ to rectangular axes through (0, 1) inclined at an angle $\frac{\pi}{4}$ to the original axes.

(b) Show that $x^2 + (\alpha\sqrt{3}y - 3)x + (3y^2 - 3\sqrt{3}y, -4) = 0$ respresents a pair of straight lines. Also find distance between mean. 3,3

2. (a) Prove that the joint equation of straight lines bisecting the angles between lines :

$$ax^2 + 24\,xy + by^2 = 0 \text{ is } \frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

(b) Find equation of pair of lines joining the origin to the points of intersection of line $y = mx + c$ with the curve $x^2 + y^2 = a^2$. Prove that they are perpendicular if $2\,c^2 = a^2(1 + m^2)$. 3, 3

3. (a) Find the locus of mid-points of the chords of the circle $x^2 + y^2 = 16$ which touch the circle $(x - 4)^2 + (y - 3)^2 = 36$.

(b) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 8\,x + 12 = 0$ and $x^2 + y^2 - 4x - 6\,y - 3 = 0$. 3,3

4. (a) The point (2, 1) is a limiting point of a coaxial system of circle of which $x^2 + y^2 - 4y - 3 = 0$ is 9 member. Find the equation of the radical axis and the co-ordinates of the other limiting point.

(b) Find the equation of circle which passes through the point (2, 0) and touches the straight line $x + 2y - 1 = 0$ at the point $(3, -3)$. 3,3

Section-B

5. (a) Prove that the locus of the middle points of the normal chords of the parabola $y^2 = 4\,ax$ is :

$$\frac{y^2}{2a} + \frac{4a^2}{y^2} = x - 2a$$

(b) Prove that in a parabola the chords of contact of tangents at eight angles passes through focus. 3,3

6. (a) Show that the minimum angle between a pair of conjugate diameter of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)$.

(b) Prove that the locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which touch the circle on the join of the foci of the ellipse as diameter is :

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=a^2e^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$$ 3,3

7. (a) Prove that the pole of $px+my=1$ w. r. t. the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ lies on the ellipse $\frac{x^2}{9a^2}+\frac{y^2}{9b^2}=1$ if $a^2p^2+b^2m^2=9$.

(b) If $y=x$ is a diameter of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and eccentricity of the ellipse is $\frac{1}{\sqrt{3}}$, find the equation of the diameter conjugate to it. 3,3

8. (a) Show that the locus of the mid-points of the chords of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ whose pole lie on the line $x+y-1=0$ is the hyperbola:

$$\frac{x^2}{16}-\frac{y^2}{9}=x+y$$

(b) Find the asymptotes of the hyperbola $xy-x-2y-5=0$. Also find the equation of the conjugate hyperbola. 3,3

MATHEMATICS Paper–II

(Calculus–I)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section-A

1. (a) Solve for x the inequality $\frac{x+2}{n-2} < \frac{4n-1}{2n-3}$.

(b) Prove that $\left|x - \frac{1}{2}\right| < \frac{1}{3}$ iff $\frac{1}{11} < \frac{1-x}{1+x} < \frac{5}{7}$. 3,3

2. (a) State order completeness property of reals. Does the set of rational numbers possess this property ? Justify your answer.

(b) Find the least upper bound and greatest lower bound of the set S = $\left\{\frac{2-x}{1-x}; x > 0, x \neq 1\right\}$. 3, 3

3. (a) Is the union of two bounded sets a bounded set ? What do you say about its converse ? Justify your answer.

(b) If $f(x) = x\left[\frac{1}{x}\right]$, does $\underset{x \to 1/2}{\text{Lt}} f(x)$ exist, explain your answer. 3,3

4. (a) Prove that if a function $f(x)$ is continuous at a point a and $f(a) \neq 0$, then prove that these exists some neighbourhood of a where $f(x)$ possesses the same sign as that if $f(a)$.

(b) Determine the values of a and b for which :

$$\underset{n \to 0}{\text{Lt}} \frac{x(1 + a\cosh x) - b\sinh x}{x^3} - 1$$ 3,3

Section-B

5. (a) By using Lagrange's mean vlaue theorem prove that :

$|\sin x - \sin y| \leq |x - y|$ for all $x, y \in R$

(b) Calculate the approximate value of $\sqrt{24}$ to three decimal places by Taylor's expansion. 3,3

6. (a) Use Maclaurin's theorem to show that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\dots\dots + \frac{(-1)^{n-1}}{n}x^n - \frac{x^n}{(1+\theta x)^n}, 0 < \theta < 1$$

(b) Use mean value theorem to show that :

$$\frac{x}{6} + \frac{2n-1}{\sqrt{3}} \le \sin^{-1} x \le \frac{x}{6} + \frac{2n-1}{2\sqrt{1-x^2}} \text{ where } \frac{1}{2} \le x < 1.$$ 3.3

7. (a) If $y = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}$, show that : $\left(\frac{dy}{dx}\right)^2 = x^2 + a^2$

(b) Prove that $\tanh^{-1} x = \frac{1}{2}\log\frac{1+x}{1-x}, -1 < x < 1$ and find its derivative also. 33

8. (a) Prove that :

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n \lfloor n}{x^{n+1}}\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} \dots\dots\dots\dots - \frac{1}{n}\right].$$

(b) If $y = \sin m(\sin^{-1} x)$, show that :

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$

Hence show that :

$$y_n(0) = \begin{cases} 0, & \text{when } n \text{ is even} \\ m(1^2 - m^2)\ (3^2 - m^2)\dots\dots\dots[(n-2)^2 - m^2] \end{cases}$$

when n is odd. 3.

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