MATHEMATICS Paper–I

(Plane Geometry)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : Attempt **five** questions in all, selecting at least **two** questions from each Section.

Section–I

1. (a) Find the transformed equation of $17x^2 - 16xy + 17y^2 - 225 = 0$ when the axes are rotated through an angle of 45°.

   (b) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x + 2h'xy + b'y^2 = 1$ represent the same conic and axes are rectangular, then $(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$. 3,3

2. (a) Prove that the angle between the lines joining the origin to the points of intersection of the straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

   (b) If p_1, p_2 be the lengths of perpendiculars drawn from point $(-1, 2)$ to the pair of lines $2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0$, then prove that $p_1 p_2 = \frac{12}{5}$. 3,3

3. (a) Find the equation of circle which passes through the point (2, 0) and touches the straight line $x + 2y - 1 = 0$ at the point $(3, -1)$.

   (b) The line $2x - y = 4$ meets the circle $x^2 + y^2 - 6x + 2y + 2 = 0$ at the points P and Q. If the tangents at P and Q meet at R. Find the coordinates of R. 3.3

4. (a) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 8x + 12 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$.

   (b) Find the limiting points of the co-axial system determined by the circles $x^2 + y^2 - 6x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 4y + 3 = 0$. 3,3

Section–II

5. (a) Prove that the locus of points such that two of the three normal from them to parabola $y^2 = 4ax$, coincide is $27ay^2 = 4(x - 2a)^3$.

   (b) Find the locus of poles of normal chord of the parabola $y^2 = 4ax$. 3,3

https://questionpaperesult.com/

Scanned by CamScanner

6. (a) Prove that the locus of the foot of the perpendicular from the focus on any tangent to a parabola is the tangent at the vertex.

(b) If the normal at any point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meet the major axes in G, find the locus of mid-point of the chord PG. 3,3

7. (a) Find the lengths of the semi-diameter conjugate to the diameter y= 3x of the ellipse $ax^2+4y^2=36$.

(b) Find the asymptotes to the hyperbola $3x^2-5xy-2y^2+5x+11y-8=0$. Also find the equation of its conjugate hyperbola. 3,3

8. (a) If e and e′ be the eccentricities of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$ respectively, then prove that $\frac{1}{e^2}-\frac{1}{e'^2}=1$.

(b) Identify the curve $x^2-4xy+4y^2-32x+4y+16=0$. Find its latus rectum, focus and directrix. 3,3

MATHEMATICS Paper–II

(Calculus–I)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : (i) Attempt **five** questions, selecting at least **two** questions from each Unit.

(ii) Each question will carry **6** marks.

Unit–I

1. (a) Solve the inequation : $\frac{2}{x-2}<\frac{x+2}{x-2}<2$.

(b) State and prove Archimedian property. Using the property prove that the set of natural numbers N is not bounded above. (3,3)

2. (a) Show that $\lim_{x\to 0}\sin\frac{1}{x}$ does not exists.

(b) Evaluate : $\lim_{x\to 1/2}\frac{1}{x}\left[\frac{1}{x}\right]$, if exists. (3,3)

https://questionpaperesult.com/

Scanned by CamScanner

3. (a) Use intermediate value theorem to show that equation six $x - x + 1 =$ 0 has a real root.

(b) Evaluate : $\lim_{x \to 0} \frac{x - \sin x}{\tan^3 x}$. (3,3)

4. (a) Evaluate : $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

(b) Discuss the continuity of $f(x) = \begin{cases} \frac{|x| + x}{3} & , \quad x \le 3 \\ \frac{2|x-3|}{x-3} & , \quad x > 3 \end{cases}$ over R.

(3,3)

Unit–II

5. (a) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t x.

(b) Let f be a real valued function defined in [a, b] such that (i) f is continuous in [a, b] (ii) f is differentiable in (a, b) (iii) f(a) = f(b), then there exists at least one CE(a, b) such that $f'(c) = 0$. (2, 4)

6. (a) Prove that $\tanh^{-1} x = \frac{1}{2} \log \left(\frac{x+1}{1-x} \right), -1 < x < 1$, and then find its derivative.

(b) Use Cauchy's mean value theorem to evaluate $\lim_{x \to 1} \frac{\cos \frac{\pi x}{2}}{\log \frac{1}{x}}$. (3,3)

7. (a) Use mean value theorem to prove :

$$\frac{x}{1+x} < \log(1+x) < x \text{ for } x > -1, x \neq 0.$$

(b) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. (3,3)

8. (a) State and prove leibnitz's Theorem.

(b) If $y = \frac{\log x}{x}$, prove that $y_n = \frac{(-1)^n \underline{|n}}{x^{n+1}} \left[\log x - 1 \frac{-1}{2} \frac{-1}{3} \ldots\ldots \frac{-1}{n} \right]$.

(3,3)

https://questionpaperesult.com/

Scanned by CamScanner

MATHEMATICS Paper–III

(Trigonometry and Matrices)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : (i) Attempt five questions in all by selecting at least two questions from each Unit.

(ii) All questions carry equal marks.

Unit–I

1. (a) Find the four 4th roots of $1-\sqrt{-3}$. 3

(b) Solve the equation :

$x^9 - x^5 + x^4 - 1 = 0.$ 3

2. (a) Show that root of equation :

$(1+x)^n - (1-x)^n = 0.$

are $i \tan\left(\frac{k\pi}{n}\right)$, $k = 0, 1, 2, 3, \ldots\ldots\ldots, n-1$. 3

(b) Expand $\cos^7\theta$ in terms of cosine of multiple of θ. 3

3. (a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, show that :

$$\phi = \frac{1}{2}\log\left[\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right].$$ 3

(b) If S_n denote the sum of n terms of the series :

$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots\ldots$ prove that :

$$\lim_{n\to\infty}\frac{1}{n}(S_1 + S_2 + S_3 + \ldots\ldots + S_n) = \frac{1}{2}\cot\left(\frac{x}{2}\right).$$ 3

4. (a) Sum to n terms the series :

$\tan^{-1}\frac{1}{3} - \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots$ and deduce the sum to infinite terms. 3

(b) If $i^{\alpha+i\beta} = \alpha + i\beta \; (\alpha, \beta \in \mathbb{R})$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$. 3

https://questionpaperesult.com/

Scanned by CamScanner

Unit–II

5. (a) Prove that a necessary and sufficient condition for a matrix A to be Hermition is that $A^{\Theta} = A$. 3

(b) Define rank of a matrix. Prove that points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ in a plane are collinear if and only if rank of the matrix :

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$$ is less than three. ½+2½

6. (a) Reduce : $A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{pmatrix}$ to normal form and hence find its rank. 3

(b) Using elementary operations, find inverse of matrix :

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$ 3

7. (a) When a system of linear equations is said to be consistant ? Find the values of λ and μ so that the system of equations :

$$2x - 3y + 5z = 12$$
$$3x + y + \lambda z = \mu$$
$$x - 7y + 8z = 17$$

has (i) a unique solution (ii) infinite solutions (iii) No solution. ½+2½

(b) State and prove Cayley Hamilton theorem. 3

8. (a) Show that the matrix $A\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is not diagonalizable over $\mathbb{R}$, however, A is diagonalizable over $\mathbb{C}$. Find an invertible matrix P over $\mathbb{C}$ such that $P^{-1}AP$ is a diagonal matrix. 1+2

(b) Prove that the modulus of each characteristic root of a unitary matrix is unity. 3

❖ ❖ ❖

https://questionpaperesult.com/

Scanned by CamScanner