

(Solid Geometry)

Time Allowed : Three Hours

Maximum Marks : 30

Note : Attempt five questions, selecting at least two questions from each section.
All questions carry equal marks.

Section - A

1. (a) Shift the origin to a suitable point so that the equation $x^2 + y^2 + z^2 - 4x - 8y + 6z - 4 = 0$ is transformed into an equation in which the first degree terms are absent.
- (b) Transform the equation $13x^2 + 13y^2 + 10z^2 + 8xy - 4yz - 4zx - 144 = 0$ when the axes are rotated to the axes having direction cosines $\left\langle -\frac{1}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$, $\left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$ and $\left\langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle$.
2. (a) Find the equation of the sphere passing through $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and whose centre lies on the plane $3x - y + z = 2$.
- (b) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 = 49$, $2x + 3y + 6z = 14$.
3. (a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.
- (b) Find the equation of the tangent planes to sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which pass through the lines $x + z - 16 = 0$, $2y - 3z + 30 = 0$.
4. (a) Find the equation of right circular cylinder of radius 3 and having for its axis the line :

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{5-z}{7}$$

- (b) Find the equation of cylinder whose generatrices are parallel to the

line $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z-3}{4}$ and whose guiding curve is the parabola $x^2 + 2y = 0, z = 0$.

Section - B

5. (a) Find the equation of the right circular cone whose vertex is at the point $(2, 1, -3)$, semivertical angle 30° and the direction cosines of whose axis are $3 : 4 : -1$.

(b) Find the equation of the quadric cone which passes through the three coordinates axes and the three mutually perpendicular lines

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}, \frac{x}{1} = \frac{y}{-1} = \frac{z}{-1}, \frac{x}{5} = \frac{y}{4} = \frac{z}{1}.$$

6. (a) Find the equation of the cone whose vector is $(2, -3, 1)$ and whose guiding curve is $4x^2 + y^2 = 1, z = 0$.
- (b) Find the equation of the cone circumscribing the sphere $x^2 + y^2 + z^2 + 2x - 2y - 2z = 0$ and having its vertex at $(1, 1, 1)$.
7. (a) Prove that the equation $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + y = 0$ represents a cone whose vector is $(-1, -2, -3)$.
- (b) Find the lines in which the plane $x - 2y - z = 0$ cuts cone $3x^2 + 4y^2 - z^2 = 0$. Find the angle between them.
8. (a) Show that the equation $x^2 + y^2 + z^2 - 6yz - 2zx - 2xy - 6x - 2y - 2z + 2 = 0$ represents a hyperboloid of two sheets.
- (b) Reduce the equation $6y^2 - 18yz - 6zx + 2xy - 9x + 5y - 5z + 2 = 0$ to the standard form.

MATHEMATICS Paper-II

(Calculus - II)

Time Allowed : Three Hours

Maximum Marks : 30

- Note:** 1. Attempt five questions in all, selecting at least two questions from each section.
2. Each question carries 6 marks.

Section - I

1. (a) Show that the line joining the two points of inflexion of the curve : $y^2(x - a) = x^2(x + a), x \neq \pm a$ subtends an angle $\pi/3$ at the origin. 3, 3
- (b) Trace the curve $y^2 = (x + 1)^3$.
2. (a) Find the asymptotes of the curve : $x^2y + xy^2 + 2x^2 - 2xy - y^2 - 6x - 2y - 2y + 2 = 0$ and show that they cut the curve in at most three points which lie on the straight line $2x - 3y - 4 = 0$.
- (b) Determine the position and nature of the double points on the curve: $x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$. 3, 3

3. (a) Define circle of curvature. Find the equation of the curve

$$\sqrt{x} + \sqrt{y} = \sqrt{a}.$$

- (b) Show that the points of intersection of the curve $xy(x^2 - y^2) - 25x^2 - 9y^2 + 144 = 0$ and its asymptotes lie on ellipse whose eccentricity is $4/5$. 3,3

4. (a) If C_o, C_p denote the lengths of chord of curvatures of the cardioid $r = a(1 + \cos\theta)$ along and perpendicular to the radius vector through any point respectively. Prove that :

$$3(C_o^2 + C_p^2) = 8aC_o.$$

- (b) Find the interval in which the curve $y = (x^2 + 4x + 5)e^{-x}$ is concave upwards or downwards. 4,2

Section – II

5. (a) If $\int_0^{\pi/4} \tan^n x dx$, show that, for $n > 1, I_n + I_{n-2} = \frac{1}{n-1}$. Hence deduce the value of I_3 .

- (b) Evaluate $\int \cosh^{-1} \left(\frac{1+x^2}{1-x^2} \right) dx$. 6

6. (a) Find the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to any point (x, y) .

- (b) Find the volume of the solid obtained by revolving the area included between the curves $y^2 = x^3$ and $x^2 = y^3$ about X-axis. 6

7. (a) Find the surface area of the solid obtained by revolving the curve y

$$= 2x + 1 + \frac{1}{x^2} \text{ about } x\text{-axis for } 1 \leq x \leq 2.$$

- (b) Use Simpson's rule with $n = 4$ to approximate $\int_{-1}^1 (x^3 + 1) dx$. Also

find the error.

3,3

8. (a) Evaluate :

$$\lim_{n \rightarrow \infty} \frac{1}{n^{16}} (1^{15} + 2^{15} + \dots + n^{15}).$$

(b) Derive the reduction formula for $\int x^n \sin(ax) dx$. Hence evaluate

$$\int_0^{\pi/2} x^3 \sin(x) dx.$$

2,4

MATHEMATICS Paper-III

(Theory of Equations)

Time Allowed : 3 Hours

Maximum Marks : 30

Note: Attempt five questions in all selecting at least two questions from each unit. All questions carry equal marks.

Unit - I

1. (a) Find a polynomial of least degree having $-2, 1, 3$ as its zeros and having value -8 at $x = 2$.
 (b) Find g.c.d. of two polynomials $f(x) = x^3 + 6x^2 + 11x + 6$ and $g(x) = x^2 + 7x + 10$. Express the g.c.d as $a(x)f(x) + b(x)g(x)$.
2. (a) Solve the equation $x^4 + 2x^3 - 2x - 1 = 0$ given that it has multiple roots.
 (b) Prove that the complex roots of a real polynomial equation occur in conjugate pairs.
3. (a) Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ given that its roots are in A.P.
 (b) Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ given that two of its roots are equal in magnitude but opposite in sign.
4. (a) Transform the equation $2x^3 - 9x^2 + 13x - 6 = 0$ into one in which second term is missing and hence solve the equation.
 (b) If α, β, γ are roots of $2x^3 + x^2 + x + 1 = 0$ form an equation whose roots are

$$\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}, \frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}, \frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}$$

Unit – II

5. (a) Find the equation whose roots are squared differences of the roots of the equation $x^3 + 6x^2 + 9x + 4 = 0$. Hence show that given equation has a double roots.
- (b) Let $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ be a real polynomial of degree n and $a_0 \neq 0$. Let r and s be the number of variations in sign of $f(x)$ and $f(-x)$ respectively. Show that $n - r - s$ is even.
6. (a) Show that the real roots of the equation $x^4 - 10x^3 - 13x^2 + 60x + 65 = 0$ lie between -4 and 12 .
- (b) Use Newton's method of divisor to find the integral roots of the equation :
 $3x^4 - 23x^3 + 35x^2 + 31x - 30 = 0$.
7. (a) Use Cardon's method to solve : $x^3 + x^2 - 16x + 20 = 0$.
- (b) For the equation $x^3 - 6x^2 - 6x - 14 = 0$, find $G^2 + 4H^3$ and hence discuss the nature of roots.
8. (a) Solve the biquadratic $x^4 - 6x^3 + 3x^2 + 22x - 6 = 0$ by Descarte's Method.
- (b) Solve by Ferrori's Method, the equation $2x^4 + 6x^3 - 3x^2 + 2 = 0$.

COMPUTER SCIENCE

Paper : CS03 : Theory - A, Operating System Concepts

Time Allowed : Three Hours

Maximum Marks : 30

Note: Attempt five questions in all, including Q-9 in Section - E, which is compulsory, taking one each from Section-A to Section - D.

Section – A

- Describe the objectives and functions of Operating System. Also explain the different services provided by the operating system. Describe the structure of an Operating System. 6
- Describe the different types of Operating Systems and their salient features with examples. 6