

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Thursday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM
PHYSICS

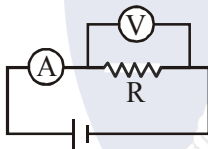
1. Two forces P and Q of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle is :
 (1) 30° (2) 60° (3) 90° (4) 120°

Ans. (4)

Sol. $4F^2 + 9F^2 + 12F^2 \cos \theta = R^2$
 $4F^2 + 36 F^2 + 24 F^2 \cos \theta = 4R^2$
 $4F^2 + 36 F^2 + 24 F^2 \cos \theta$
 $= 4(13F^2 + 12F^2 \cos \theta) = 52 F^2 + 48F^2 \cos \theta$

$$\cos \theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$$

2. The actual value of resistance R, shown in the figure is 30Ω . This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :



- (1) 350Ω (2) 570Ω (3) 35Ω (4) 600Ω

Ans. (2)

Sol. $0.95 R = \frac{R R_v}{R + R_v}$

$$0.95 \times 30 = 0.05 R_v$$

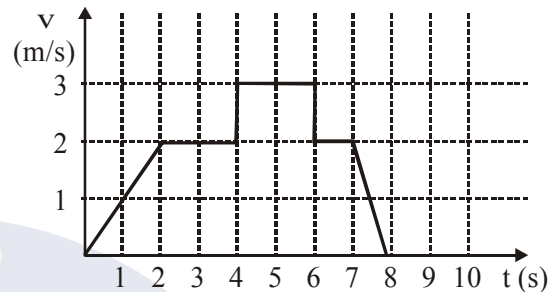
$$R_v = 19 \times 30 = 570 \Omega$$

3. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water a temperature of 8.4°C . Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C (Specific heat of brass is $394 \text{ J kg}^{-1} \text{ K}^{-1}$)
 (1) $1232 \text{ J kg}^{-1} \text{ K}^{-1}$ (2) $458 \text{ J kg}^{-1} \text{ K}^{-1}$
 (3) $654 \text{ J kg}^{-1} \text{ K}^{-1}$ (4) $916 \text{ J kg}^{-1} \text{ K}^{-1}$

Ans. (4)

Sol. $192 \times S \times (100 - 21.5)$
 $= 128 \times 394 \times (21.5 - 8.4)$
 $+ 240 \times 4200 \times (21.5 - 8.4)$
 $\Rightarrow S = 916$

4. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5\text{s}$?



- (1) 6 m (2) 9 m (3) 3 m (4) 10 m

Ans. (2)

S = Area under graph

$$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9 \text{ m}$$

5. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is :

- (1) 437.5 J (2) 637.5 J
 (3) 740 J (4) 540 J

Ans. (1)

$$L \frac{di}{dt} = 25$$

$$L \times \frac{15}{1} = 25$$

$$L = \frac{5}{3} \text{ H}$$

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5 \text{ J}$$

6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :

- (1) $11 \times 10^{-5} \text{ W}$ (2) $11 \times 10^{-4} \text{ W}$
 (3) $11 \times 10^5 \text{ W}$ (4) $11 \times 10^{-3} \text{ W}$

Ans. (1)

$$P = I^2 R$$

$$4.4 = 4 \times 10^{-6} R$$

$$R = 1.1 \times 10^6 \Omega$$

$$P' = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5} W$$

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

(1) 4260 ± 80 cm³ (2) 4300 ± 80 cm³

(3) 4264.4 ± 81.0 cm³ (4) 4264 ± 81 cm³

Ans. (1)

$$V = \pi \frac{d^2}{4} h = 4260 \text{ cm}^3$$

$$\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h}$$

$$\Delta V = 2 \times \frac{0.1V}{12.6} + \frac{0.1V}{34.2}$$

$$= \frac{0.2}{12.6} \times 4260 + \frac{0.1 \times 4260}{34.2} = 80$$

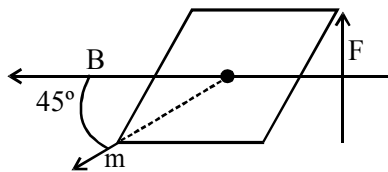
$$\text{Volume} = 4260 \pm 80 \text{ cm}^3$$

8. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N

(3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

Ans. (2)



$$mB \sin 45^\circ = F \frac{l}{2} \sin 45^\circ$$

$$F = 2mB = 3.6 \times 18 \times 10^{-6}$$

$$= 6.5 \times 10^{-5} \text{ N}$$

9. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(1) 2750 kHz (2) 2000 kHz

(3) 2250 kHz (4) 2900 kHz

Ans. (2)

$$f_{\text{carrier}} = \frac{250}{0.1} = 2500 \text{ KHZ}$$

\therefore Range of signal = 2250 Hz to 2750 Hz

Now check all options : for 2000 KHZ

$$f_{\text{mod}} = 200 \text{ Hz}$$

\therefore Range = 1800 KHZ to 2200 KHZ

10. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then :

(1) $T_h = 0.5 T_c$ (2) $T_h = 2 T_c$

(3) $T_h = 1.5 T_c$ (4) $T_h = T_c$

Ans. (4)

$$T = 2\pi \sqrt{\frac{I}{\mu B}}$$

$$T_h = 2\pi \sqrt{\frac{mR^2}{(2\mu)B}}$$

$$T_c = 2\pi \sqrt{\frac{1/2 mR^2}{\mu B}}$$

11. The electric field of a plane polarized electromagnetic wave in free space at time $t=0$ is given by an expression

$$\vec{E}(x,y) = 10\hat{j} \cos [(6x + 8z)]$$

The magnetic field \vec{B} (x, z, t) is given by : (c is the velocity of light)

(1) $\frac{1}{c}(6\hat{k} + 8\hat{i}) \cos [(6x - 8z + 10ct)]$

(2) $\frac{1}{c}(6\hat{k} - 8\hat{i}) \cos [(6x + 8z - 10ct)]$

(3) $\frac{1}{c}(6\hat{k} + 8\hat{i}) \cos [(6x + 8z - 10ct)]$

(4) $\frac{1}{c}(6\hat{k} - 8\hat{i}) \cos [(6x + 8z + 10ct)]$

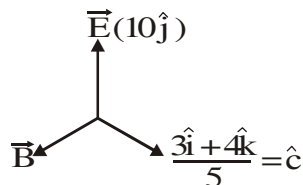
Ans. (2)

$$\vec{E} = 10\hat{j} \cos \left[(6\hat{i} + 8\hat{k}) \cdot (x\hat{i} + z\hat{k}) \right]$$

$$= 10\hat{j} \cos[\vec{K} \cdot \vec{r}]$$

$\therefore \vec{K} = 6\hat{i} + 8\hat{k}$; direction of waves travel.

i.e. direction of 'c'.



\therefore Direction of \hat{B} will be along

$$\hat{C} \times \hat{E} = \frac{-4\hat{i} + 3\hat{k}}{5}$$

Mag. of \vec{B} will be along $\hat{C} \times \hat{E} = \frac{-4\hat{i} + 3\hat{k}}{5}$

$$\text{Mag. of } \vec{B} = \frac{E}{C} = \frac{10}{C}$$

$$\therefore \vec{B} = \frac{10}{C} \left(\frac{-4\hat{i} + 3\hat{k}}{5} \right) = \frac{(-8\hat{i} + 6\hat{k})}{C}$$

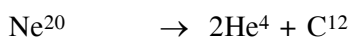
12. Consider the nuclear fission



Given that the binding energy/nucleon of Ne^{20} , He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement :

- (1) 8.3 MeV energy will be released
- (2) energy of 12.4 MeV will be supplied
- (3) energy of 11.9 MeV has to be supplied
- (4) energy of 3.6 MeV will be released

Ans. (3)



$$8.03 \times 20 \quad 2 \times 7.07 \times 4 + 7.86 \times 12$$

$$\therefore E_B = (\text{BE})_{\text{react}} - (\text{BE})_{\text{product}} = 9.72 \text{ MeV}$$

13. Two vectors \vec{A} and \vec{B} have equal magnitudes.

The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the

magnitude of $(\vec{A} - \vec{B})$. The angle between \vec{A}

and \vec{B} is :

$$(1) \sin^{-1} \left[\frac{n^2 - 1}{n^2 + 1} \right] \quad (2) \cos^{-1} \left[\frac{n - 1}{n + 1} \right]$$

$$(3) \cos^{-1} \left[\frac{n^2 - 1}{n^2 + 1} \right] \quad (4) \sin^{-1} \left[\frac{n - 1}{n + 1} \right]$$

Ans. (3)

$$|\vec{A} + \vec{B}| = 2a \cos \theta / 2 \quad \text{---(1)}$$

$$|\vec{A} - \vec{B}| = 2a \cos \frac{(\pi - \theta)}{2} = 2a \sin \theta / 2 \quad \text{---(2)}$$

$$\Rightarrow n \left(2a \sin \frac{\theta}{2} \right) = 2a \frac{\cos \theta}{2}$$

$$\Rightarrow \tan \frac{\theta}{2} = \frac{1}{n} \quad \cos \theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)}$$

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration.

Then, its periodic time in seconds is :

$$(1) \frac{7}{3}\pi \quad (2) \frac{3}{8}\pi$$

$$(3) \frac{4\pi}{3} \quad (4) \frac{8\pi}{3}$$

Ans. (4)

$$v = \omega \sqrt{A^2 - x^2} \quad \text{---(1)}$$

$$a = -\omega^2 x \quad \text{---(2)}$$

$$|v| = |a| \quad \text{---(3)}$$

$$\omega \sqrt{A^2 - x^2} = \omega^2 x$$

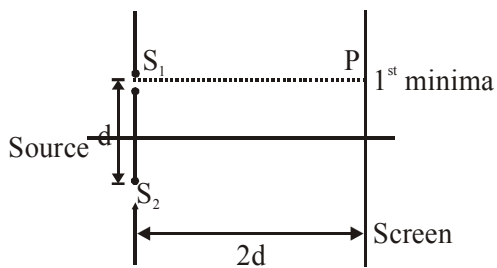
$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2 (4^2)$$

$$\Rightarrow 3 = \omega \times 4$$

$$T = 2\pi/\omega$$

15. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



- (1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{(5-\sqrt{2})}$
 (3) $\frac{\lambda}{(\sqrt{5}-2)}$ (4) $\frac{\lambda}{2(\sqrt{5}-2)}$

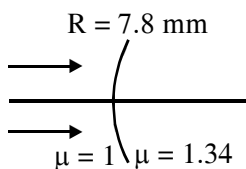
Ans. (4)

$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

16. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

- (1) 2 cm (2) 1 cm
 (3) 3.1 cm (4) 4.0 cm

Ans. (3)



$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34-1}{7.8}$$

$$\therefore V = 30.7 \text{ mm}$$

17. Half mole of an ideal monoatomic gas is heated at constant pressure of 1atm from 20 °C to 90°C. Work done by gas is close to : (Gas constant $R = 8.31 \text{ J/mol.K}$)

- (1) 73 J (2) 291 J (3) 581 J (4) 146 J

Ans. (2)

$$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$$

18. A metal plate of area $1 \times 10^{-4} \text{ m}^2$ is illuminated by a radiation of intensity 16 mW/m^2 . The work function of the metal is 5eV . The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$]

- (1) 10^{10} and 5 eV (2) 10^{14} and 10 eV
 (3) 10^{12} and 5 eV (4) 10^{11} and 5 eV

Ans. (4)

Maximum kinetic energy $KE_{\text{max}} = E - \phi$

$$KE_{\text{max}} = 10\text{eV} - 5\text{eV} = 5\text{eV}$$

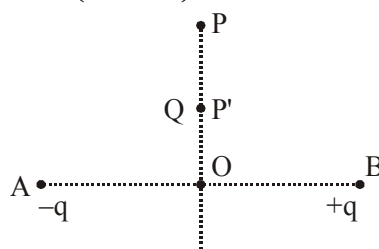
No. of photons incident per unit time $\frac{n}{t} = \frac{IA}{E}$

$$\frac{n}{t} = \frac{16 \times 10^{-3} \times 10^{-4}}{10 \times 1.6 \times 10^{-19}} = 10^{12}$$

No. of photoelectrons ejected per unit time $\frac{n}{t} = \frac{10}{100} \times 10^{12} = 10^{11}$

19. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where $OP = y$ and $y \gg 2a$. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line

to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3} \gg 2a\right)$



- (1) $\frac{F}{3}$ (2) $3F$ (3) $9F$ (4) $27F$

Ans. (4)

Sol. Electric field of equatorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$$\therefore \text{At P, } F = -\frac{K\vec{P}}{r^3}Q.$$

$$\text{At P}^1, F^1 = -\frac{K\vec{P}Q}{(r/3)^3} = 27F.$$

20. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$)

- (1) 1.4×10^5 m/s (2) 24×10^4 m/s
 (3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

Ans. (4)

By energy conservation between 0 & ∞ .

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteorite)

$$\Rightarrow v = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 \text{ m/s}$$

21. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

- (1) 7 (2) 5 (3) 6 (4) 4

Ans. (3)

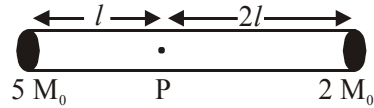
Sol. For closed organ pipe, resonant frequency is odd multiple of fundamental frequency.

$$\therefore (2n + 1) f_0 \leq 20,000$$

(f_0 is fundamental frequency = 1.5 kHz)

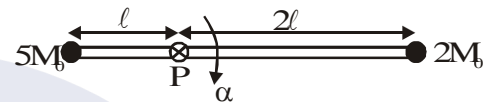
$$\therefore n = 6$$

22. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



- (1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$ (3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

Ans. (3)



Applying torque equation about point P.

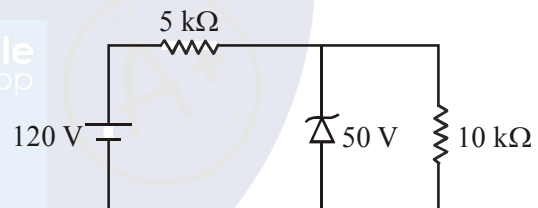
$$2M_0(2l) - 5M_0gl = I\alpha$$

$$I = 2M_0(2l)^2 + 5M_0l^2 = 13M_0l^2$$

$$\therefore \alpha = -\frac{M_0gl}{13M_0l^2} \Rightarrow \alpha = -\frac{g}{13l}$$

$$\therefore \alpha = \frac{g}{13l} \text{ anticlockwise}$$

23. For the circuit shown below, the current through the Zener diode is :



- (1) 5 mA (2) Zero (3) 14 mA (4) 9 mA

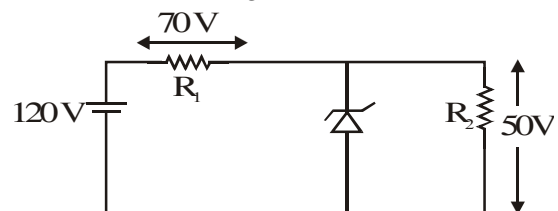
Ans. (4)

Assuming zener diode does not undergo

$$\text{breakdown, current in circuit} = \frac{120}{15000} = 8 \text{ mA}$$

$$\therefore \text{Voltage drop across diode} = 80 \text{ V} > 50 \text{ V.}$$

The diode undergoes breakdown.



$$\text{Current in } R_1 = \frac{70}{5000} = 14 \text{ mA}$$

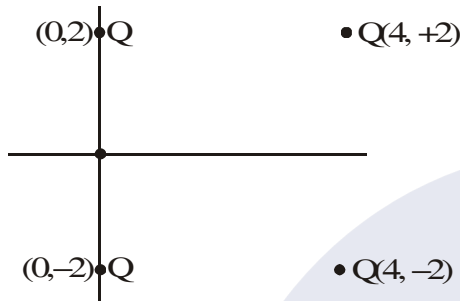
$$\text{Current in } R_2 = \frac{50}{10000} = 5 \text{ mA}$$

$$\therefore \text{Current through diode} = 9 \text{ mA}$$

24. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, -2) and (0, -2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

- (1) $\frac{Q^2}{2\sqrt{2}\pi\epsilon_0}$ (2) $\frac{Q^2}{4\pi\epsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$
 (3) $\frac{Q^2}{4\pi\epsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\epsilon_0}$

Ans. (2)



Potential at origin = $\frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$

(Potential at $\infty = 0$)

= $KQ\left(1+\frac{1}{\sqrt{5}}\right)$

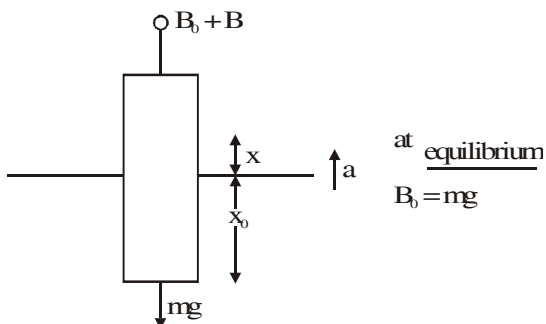
∴ Work required to put a fifth charge Q at origin

is equal to $\frac{Q^2}{4\pi\epsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

25. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω . If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3)

- (1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1}
 (3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1}

Ans. (Bonus)



Extra Boyant force = δAxg

$B_0 + B \times mg = ma$

$B = ma$

$a = \left(\frac{\delta Ag}{m}\right)^x$

$w^2 = \frac{\delta Ag}{m}$

$w = \sqrt{\frac{10^3 \times \pi(2.5)^2 \times 10^{-4} \times 10}{310 \times 10^{-6} \times 10^3}}$
 $= \sqrt{63.30} = 7.95$

26. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

- (1) 692 pJ (2) 60 pJ
 (3) 508 pJ (4) 560 pJ

Ans. (3)

$W = -(U_f - U_i)$

$= -\left(\frac{(\epsilon C)^2}{2KC} - \frac{(\epsilon C)^2}{2C}\right)$

$= \frac{\epsilon^2 C}{2} \left(\frac{K-1}{K}\right)$

$= \frac{10^2 \times 12 \times 10^{-12}}{2} \left(\frac{5.5}{6.5}\right) = 508 \text{ pJ}$

27. Two kg of a monoatomic gas is at a pressure of $4 \times 10^4 \text{ N/m}^2$. The density of the gas is 8 kg / m^3 . What is the order of energy of the gas due to its thermal motion ?

- (1) 10^3 J (2) 10^5 J
 (3) 10^6 J (4) 10^4 J

Ans. (4)

Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right)$

$$= \frac{N}{N_A} \frac{3}{2} RT$$

$$= \frac{3}{2} (nRT)$$

$$= \frac{3}{2} PV$$

$$= \frac{3}{2} P \left(\frac{m}{8} \right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8}$$

$$= 1.5 \times 10^4$$

order will 10^4

28. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of

$\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

- (1) 15 J (2) 10 J (3) 12 J (4) 9 J

Ans. (1)

$$\begin{aligned} \text{Work done} &= \vec{F} \cdot \vec{d} \\ &= 12J \end{aligned}$$

work energy theorem

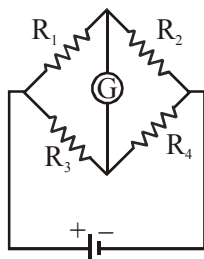
$$W_{\text{net}} = \Delta K.E.$$

$$12 = K_f - 3$$

$$K_f = 15J$$

29. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω , respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3 , would be :



- (1) Red, Green, Brown
 (2) Brown, Blue, Brown
 (3) Grey, Black, Brown
 (4) Brown, Blue, Black

Ans. (2)

$R_1 = 32 \times 10 = 320$
 for wheat stone bridge

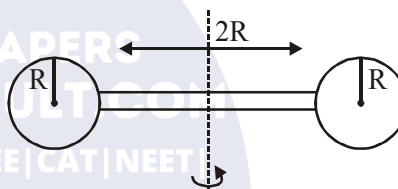
$$\Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$$R_3 = 160$$

Brown Blue Brown

30. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15} MR^2$

(2) $\frac{17}{15} MR^2$

(3) $\frac{137}{15} MR^2$

(4) $\frac{209}{15} MR^2$

Ans. (3)

For Ball
 using parallel axis theorem.

$$I_{\text{ball}} = \frac{2}{5} MR^2 + M(2R)^2$$

$$= \frac{22}{5} MR^2$$

2 Balls so $\frac{44}{5} MR^2$

Irod = for rod $\frac{M(2R)^2}{R} = \frac{MR^2}{3}$

$$I_{\text{system}} = I_{\text{Ball}} + I_{\text{rod}}$$

$$= \frac{44}{5} MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15} MR^2$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Thursday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM
CHEMISTRY

1. An ideal gas undergoes isothermal compression from 5 m³ against a constant external pressure of 4 Nm⁻². Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J mol⁻¹ K⁻¹, the temperature of Al increases by :

- (1) $\frac{3}{2}$ K (2) $\frac{2}{3}$ K (3) 1 K (4) 2 K

Ans. (2)

Sol. Work done on isothermal irreversible for ideal gas

$$= -P_{\text{ext}}(V_2 - V_1)$$

$$= -4 \text{ N/m}^2 (1\text{m}^3 - 5\text{m}^3)$$

$$= 16 \text{ Nm}$$

Isothermal process for ideal gas

$$\Delta U = 0$$

$$q = -w$$

$$= -16 \text{ Nm}$$

$$= -16 \text{ J}$$

Heat used to increase temperature of Al

$$q = n C_m \Delta T$$

$$16 \text{ J} = 1 \times 24 \frac{\text{J}}{\text{mol.K}} \times \Delta T$$

$$\Delta T = \frac{2}{3} \text{ K}$$

2. The 71st electron of an element X with an atomic number of 71 enters into the orbital :

- (1) 4f (2) 6p (3) 6s (4) 5d

Ans. (1)

3. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B₂H₆, respectively, are :

- (1) 2 and 4 (2) 2 and 1
 (3) 2 and 2 (4) 4 and 2

Ans. (4)

4. The amount of sugar (C₁₂H₂₂O₁₁) required to prepare 2 L of its 0.1 M aqueous solution is :

- (1) 68.4 g (2) 17.1 g (3) 34.2 g (4) 136.8 g

Ans. (1)

Sol. Molarity = $\frac{(n)_{\text{solute}}}{V_{\text{solution}} \text{ (in lit)}}$

$$0.1 = \frac{\text{wt.}/342}{2}$$

$$\text{wt (C}_{12}\text{H}_{22}\text{O}_{11}) = 68.4 \text{ gram}$$

5. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is :

- (1) H₂ + I₂ → 2HI (2) H₂ + F₂ → 2HF
 (3) H₂ + Cl₂ → 2HCl (4) H₂ + Br₂ → 2HBr

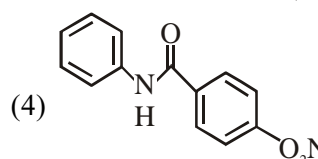
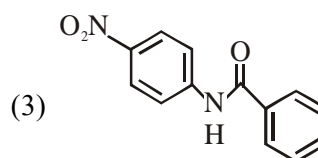
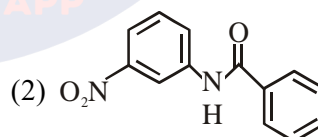
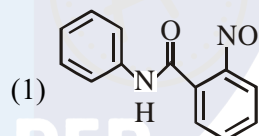
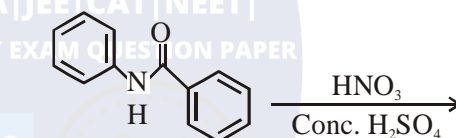
Ans. (1)

6. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

- (1) sodium ion-ammonia complex
 (2) sodamide
 (3) sodium-ammonia complex
 (4) ammoniated electrons

Ans. (4)

7. What will be the major product in the following mononitration reaction?



Ans. (3)

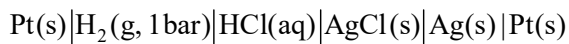
Sol. amine is o-p directing

8. In the cell $\text{Pt(s)}|\text{H}_2(\text{g}, 1\text{bar})|\text{HCl}(\text{aq})|\text{Ag(s)}|\text{Pt(s)}$ the cell potential is 0.92 when a 10^{-6} molal HCl solution is used. The standard electrode potential of $(\text{AgCl}/\text{Ag}, \text{Cl}^-)$ electrode is :

$$\left\{ \text{given, } \frac{2.303RT}{F} = 0.06\text{V at } 298\text{K} \right\}$$

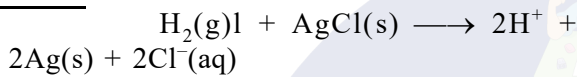
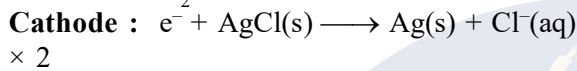
- (1) 0.20 V (2) 0.76 V (3) 0.40 V (4) 0.94 V

Ans. (1)



Sol.

$$10^{-6} \text{ m}$$



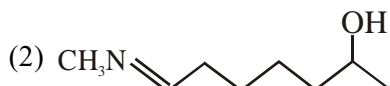
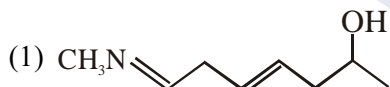
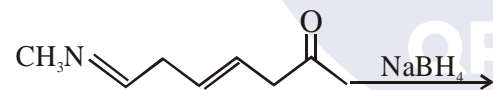
$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.06}{2} \log_{10} ((\text{H}^+)^2 \cdot (\text{Cl}^-)^2)$$

$$.925 = \left(E_{\text{H}_2/\text{H}^+}^0 + E_{\text{AgCl}/\text{Ag}, \text{Cl}^-}^0 \right) - \frac{0.06}{2} \log_{10} ((10^{-6})^2 (10^{-6})^2)$$

$$.92 = 0 + E_{\text{AgCl}/\text{Ag}, \text{Cl}^-}^0 - 0.03 \log_{10} (10^{-6})^4$$

$$E_{\text{AgCl}/\text{Ag}, \text{Cl}^-}^0 = .92 + .03 \times -24 = 0.2 \text{ V}$$

9. The major product of the following reaction is:



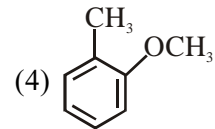
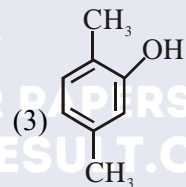
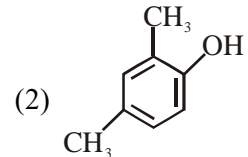
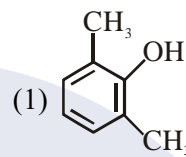
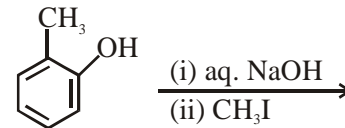
Ans. (3)

10. The pair that contains two P-H bonds in each of the oxoacids is :

- (1) H_3PO_2 and $\text{H}_4\text{P}_2\text{O}_5$
 (2) $\text{H}_4\text{P}_2\text{O}_5$ and $\text{H}_4\text{P}_2\text{O}_6$
 (3) H_3PO_3 and H_3PO_2
 (4) $\text{H}_4\text{P}_2\text{O}_5$ and H_3PO_3

Ans. (1)

11. The major product of the following reaction is:



Ans. (4)

Sol. S_{N}^2 reaction

12. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is :

- (1) Fe^{2+} (2) Co^{2+} (3) Mn^{2+} (4) Ni^{2+}

Ans. (2)

Sol. $\text{Co}^{2+} \rightarrow d^7$
 hs, $n = 3$, ls, $n = 1$

13. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms :

- (1) hcp lattice-A, $\frac{2}{3}$ Tetrahedral voids-B
 (2) hcp lattice-B, $\frac{1}{3}$ Tetrahedral voids-A
 (3) hcp lattice-B, $\frac{2}{3}$ Tetrahedral voids-A
 (4) hcp lattice-A $\frac{1}{3}$ Tetrahedral voids-B

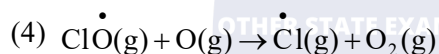
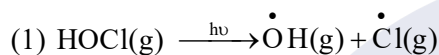
Ans. (2)

Sol. A_2B_3 has HCP lattice

If A form HCP, then $\frac{3}{4}$ of THV must occupied by B to form A_2B_3

If B form HCP, then $\frac{1}{3}$ of THV must occupied by A to form A_2B_3

14. The reaction that is NOT involved in the ozone layer depletion mechanism in the stratosphere is:



Ans. (3)

Sol. Conceptual

15. The process with negative entropy change is :

(1) Dissolution of iodine in water

(2) Synthesis of ammonia from N_2 and H_2

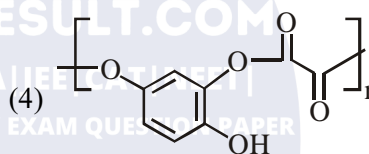
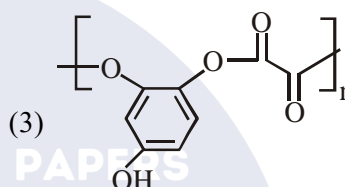
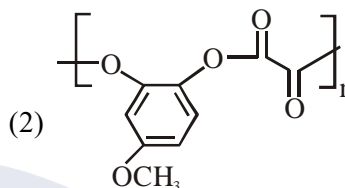
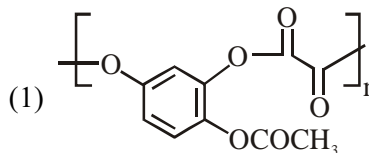
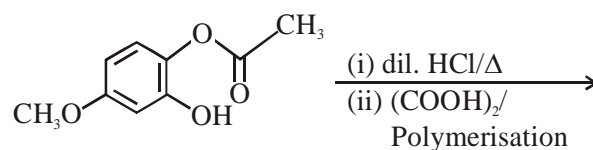
(3) Dissolution of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$

(4) Sublimation of dry ice

Ans. (2)

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$; $\Delta n_g < 0$

16. The major product of the following reaction is:



Ans. (3)

17. A reaction of cobalt(III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) B (green coloured). A can show optical activity, B is optically inactive. What type of isomers does A and B represent ?

(1) Geometrical isomers

(2) Ionisation isomers]

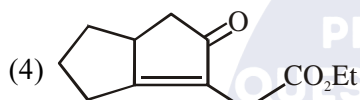
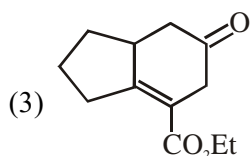
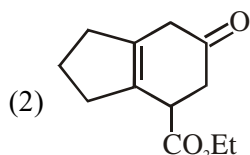
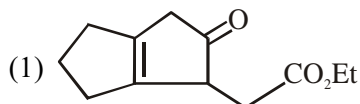
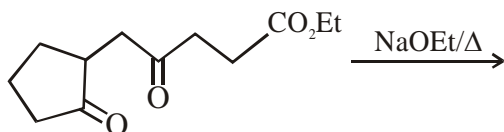
(3) Coordination isomers

(4) Linkage isomers

Ans. (1)

Sol. $[Co(en)_2Cl_2]Cl$
cis --> Optically active
trans --> Optically inactive

18. The major product obtained in the following reaction is :



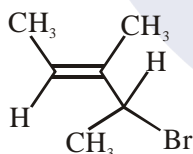
Ans. (4)

19. Which of the following tests cannot be used for identifying amino acids ?

- (1) Biuret test (2) Xanthoproteic test
(3) Barfoed test (4) Ninhydrin test

Ans. (3)

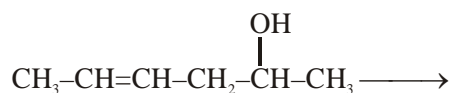
20. What is the IUPAC name of the following compound ?



- (1) 3-Bromo-1, 2-dimethylbut-1-ene]
(2) 4-Bromo-3-methylpent-2-ene
(3) 2-Bromo-3-methylpent-3-ene
(4) 3-Bromo-3-methyl-1, 2-dimethylprop-1-ene

Ans. (2)

21. Which is the most suitable reagent for the following transformation ?



- (1) alkaline KMnO_4 (2) I_2/NaOH
(3) Tollen's reagent (4) CrO_2/CS_2

Ans. (2)

22. The correct match between item 'I' and item 'II' is :

| Item 'I' (compound) | Item 'II' (reagent) |
|------------------------|----------------------------|
| (A) Lysine | (P) 1-naphthol |
| (B) Furfural | (Q) ninhydrin |
| (C) Benzyl alcohol | (R) KMnO_4 |
| (D) Styrene | (S) Ceric ammonium nitrate |

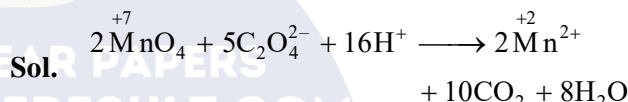
- (1) (A)→(Q), (B)→(P), (C)→(S), (D)→(R)
(2) (A)→(Q), (B)→(R), (C)→(S), (D)→(P)
(3) (A)→(Q), (B)→(P), (C)→(R), (D)→(S)
(4) (A)→(R), (B)→(P), (C)→(Q), (D)→(S)

Ans. (1)

23. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is :

- (1) 10 (2) 2 (3) 1 (4) 5

Ans. (3)

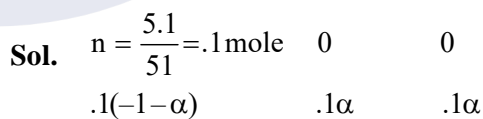
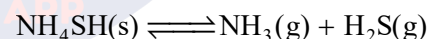


10 e^- trans for 10 molecules of CO_2 so per molecule of CO_2 transfer of e^- is '1'

24. 5.1g NH_4SH is introduced in 3.0 L evacuated flask at 327°C . 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327°C is ($R = 0.082 \text{ L atm mol}^{-1}\text{K}^{-1}$, Molar mass of S = 32 g mol^{-1} , molar mass of N = 14 g mol^{-1})

- (1) $1 \times 10^{-4} \text{ atm}^2$ (2) $4.9 \times 10^{-3} \text{ atm}^2$
(3) 0.242 atm^2 (4) $0.242 \times 10^{-4} \text{ atm}^2$

Ans. (3)



$$\alpha = 30\% = .3$$

so number of moles at equilibrium

$$\begin{array}{ccc} .1(1-.3) & .1 \times .3 & .1 \times .3 \\ = & .07 & =.03 & =.03 \end{array}$$

Now use $PV = nRT$ at equilibrium

$$P_{\text{total}} \times 3 \text{ lit} = (.03 + .03) \times .082 \times 600$$

$$P_{\text{total}} = .984 \text{ atm}$$

At equilibrium

$$P_{\text{NH}_3} = P_{\text{H}_2\text{S}} = \frac{P_{\text{total}}}{2} = .492$$

$$\text{So } k_p = P_{\text{NH}_3} \cdot P_{\text{H}_2\text{S}} = (.492) (.492)$$

$$k_p = .242 \text{ atm}^2$$

25. The electrolytes usually used in the electroplating of gold and silver, respectively, are :

- (1) $[\text{Au}(\text{OH})_4]^-$ and $[\text{Ag}(\text{OH})_2]^-$
- (2) $[\text{Au}(\text{CN})_2]^-$ and $[\text{Ag} \text{Cl}_2]^-$
- (3) $[\text{Au}(\text{NH}_3)_2]^+$ and $[\text{Ag}(\text{CN})_2]^-$
- (4) $[\text{Au}(\text{CN})_2]^-$ and $[\text{Ag}(\text{CN})_2]^-$

Ans. (4)

26. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point of 2 molal solutions of glucose in the same solvent is 2 K. The relation between K_b and K_f is:

- (1) $K_b = 0.5 K_f$
- (2) $K_b = 2 K_f$
- (3) $K_b = 1.5 K_f$
- (4) $K_b = K_f$

Ans. (2)

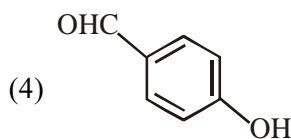
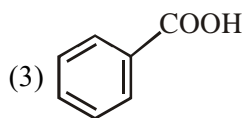
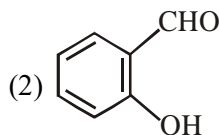
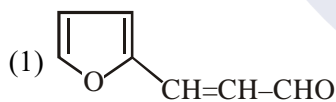
Sol. Ans.(2)

$$\frac{\Delta T_b}{\Delta T_f} = \frac{i \cdot m \times k_b}{i \cdot m \times k_f}$$

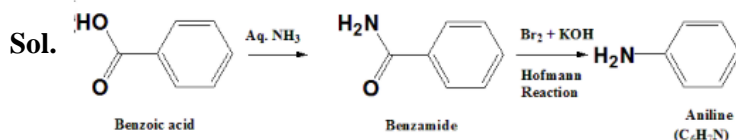
$$\frac{2}{2} = \frac{1 \times 1 \times k_b}{1 \times 2 \times k_f}$$

$$k_b = 2k_f$$

27. An aromatic compound 'A' having molecular formula $\text{C}_7\text{H}_6\text{O}_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula $\text{C}_6\text{H}_7\text{N}$. The structure of 'A' is :



Ans. (3)



28. The ground state energy of hydrogen atom is -13.6 eV . The energy of second excited state He^+ ion in eV is :

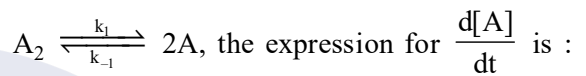
- (1) -6.04
- (2) -27.2
- (3) -54.4
- (4) -3.4

Ans. (1)

Sol. $(E)_n^{\text{th}} = (E_{\text{GND}})_H \cdot \frac{Z^2}{n^2}$

$$E_{3^{\text{rd}}}(\text{He}^+) = (-13.6 \text{ eV}) \cdot \frac{2^2}{3^2} = -6.04 \text{ eV}$$

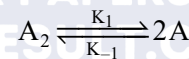
29. For an elementary chemical reaction,



- (1) $2k_1[\text{A}_2] - k_{-1}[\text{A}]^2$
- (2) $k_1[\text{A}_2] - k_{-1}[\text{A}]^2$
- (3) $2k_1[\text{A}_2] - 2k_{-1}[\text{A}]^2$
- (4) $k_1[\text{A}_2] + k_{-1}[\text{A}]^2$

Ans. (3)

Sol. Ans.(3)



$$\frac{d[\text{A}]}{dt} = 2k_1[\text{A}_2] - 2k_{-1}[\text{A}]^2$$

30. Haemoglobin and gold sol are examples of :

- (1) negatively charged sols
- (2) positively charged sols]
- (3) negatively and positively charged sols, respectively
- (4) positively and negatively charged sols, respectively

Ans. (4)

Sol. Ans.(4)

Haemoglobin \longrightarrow positive sol

Ag - sol \longrightarrow negative sol

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM
MATHEMATICS

1. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I[z]$

respectively denote the real and imaginary parts of z , then :

- (1) $R(z) > 0$ and $I(z) > 0$
- (2) $R(z) < 0$ and $I(z) > 0$
- (3) $R(z) = -3$
- (4) $I(z) = 0$

Ans. (4)

Sol. $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$

$$z = (e^{i\pi/6})^5 + (e^{-i\pi/6})^5$$

$$= e^{i5\pi/6} + e^{-i5\pi/6}$$

$$= \cos \frac{5\pi}{6} + i \frac{\sin 5\pi}{6} + \cos \left(\frac{-5\pi}{6}\right) + i \sin \left(\frac{-5\pi}{6}\right)$$

$$= 2 \cos \frac{5\pi}{6} < 0$$

$I(z) = 0$ and $Re(z) < 0$

Option (4)

2. Let $a_1, a_2, a_3, \dots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \dots, 10$ and S be the set of pairs (r, k) , $r, k \in \mathbb{N}$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S , is :

- (1) Infinitely many
- (2) 4
- (3) 10
- (4) 2

Ans. (1)

Sol. Apply

$$C_3 \rightarrow C_3 - C_2$$

$$C_2 \rightarrow C_2 - C_1$$

We get $D = 0$

Option (1)

3. The positive value of λ for which the co-efficient of x^2 in the expression

$$x^2 \left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$$

is 720, is :

- (1) $\sqrt{5}$
- (2) 4
- (3) $2\sqrt{2}$
- (4) 3

Ans. (2)

Sol. $x^2 \left({}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{\lambda}{x^2}\right)^r \right)$

$$x^2 \left[{}^{10}C_r (x)^{\frac{10-r}{2}} (\lambda)^r (x)^{-2r} \right]$$

$$x^2 \left[{}^{10}C_r \lambda^r x^{\frac{10-5r}{2}} \right]$$

$\therefore r = 2$

Hence, ${}^{10}C_2 \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

4. The value of $\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$ is :

- (1) $\frac{1}{256}$
- (2) $\frac{1}{2}$
- (3) $\frac{1}{512}$
- (4) $\frac{1}{1024}$

Ans. (3)

Sol. $2 \sin \frac{\pi}{2^{10}} \cos \frac{\pi}{2^{10}} \cdot \dots \cdot \cos \frac{\pi}{2^2}$

$$\frac{1}{2^9} \sin \frac{\pi}{2} = \frac{1}{512}$$

Option (3)

5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where $[t]$

denotes the greatest integer less than or equal to t , is :

(1) $\frac{1}{12}(7\pi + 5)$ (2) $\frac{3}{10}(4\pi - 3)$

(3) $\frac{1}{12}(7\pi - 5)$ (4) $\frac{3}{20}(4\pi - 3)$

Ans. (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$

$= \int_{-\pi/2}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^0 \frac{dx}{-1-1+4}$

$+ \int_0^1 \frac{dx}{0+0+4} + \int_1^{\pi/2} \frac{dx}{1+0+4}$

$\int_{-\pi/2}^{-1} \frac{dx}{1} + \int_{-1}^0 \frac{dx}{2} + \int_0^1 \frac{dx}{4} + \int_1^{\pi/2} \frac{dx}{5}$

$\left(-1 + \frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$

$-1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{\pi}{2} + \frac{\pi}{10}$

$\frac{-20+10+5-4}{20} + \frac{6\pi}{10}$

$\frac{-9}{20} + \frac{3\pi}{5}$

Option (4)

6. If the probability of hitting a target by a shooter, in any shot, is $1/3$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target

at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $1 - {}^n C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$

$n_{\min} = 5 \Rightarrow$ Option (2)

7. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, \dots, x_5 and -50 is equal to :

(1) 582.5 (2) 507.5

(3) 586.5 (4) 509.5

Ans. (2)

Sol. $\bar{x} = 10 \Rightarrow \sum_{i=1}^5 x_i = 50$

S.D. = $\sqrt{\frac{\sum_{i=1}^5 x_i^2}{5} - (\bar{x})^2} = 8$

$\Rightarrow \sum_{i=1}^5 (x_i)^2 = 109$

variance = $\frac{\sum_{i=1}^5 (x_i)^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^5 x_i - 50}{6}\right)^2$
 $= 507.5$

Option (2)

8. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. $x^2 = 4y$

$$x - \sqrt{2}y + 4\sqrt{2} = 0$$

Solving together we get

$$x^2 = 4 \left(\frac{x + 4\sqrt{2}}{\sqrt{2}} \right)$$

$$\sqrt{2}x^2 + 4x + 16\sqrt{2}$$

$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$

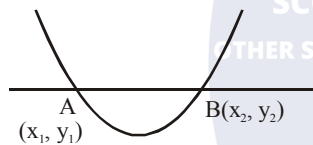
$$x_1 + x_2 = 2\sqrt{2}; \quad x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$$

Similarly,

$$(\sqrt{2}y - 4\sqrt{2})^2 = 4y$$

$$2y^2 + 32 - 16y = 4y$$

$$2y^2 - 20y + 32 = 0 \begin{cases} y_1 + y_2 = 10 \\ y_1 y_2 = 16 \end{cases}$$



$$\begin{aligned} l_{AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2\sqrt{2})^2 + 64 + (10)^2 - 4(16)} \\ &= \sqrt{8 + 64 + 100 - 64} \\ &= \sqrt{108} = 6\sqrt{3} \end{aligned}$$

Option (3)

9. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the

minimum value of $\frac{\det(A)}{b}$ is :

- (1) $\sqrt{3}$ (2) $-\sqrt{3}$
 (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (4)

Sol. $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ ($b > 0$)

$$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$$

$$|A| = 2(b^2 + 2) - b^2 - 1$$

$$|A| = b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b + \frac{3}{b}}{2} \geq \sqrt{3}$$

$$b + \frac{3}{b} \geq 2\sqrt{3}$$

Option (4)

10. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\frac{4}{3}, 2e\right)$ (2) (2,3e)

(3) $\left(\frac{5}{3}, 2e\right)$ (4) (3,6e)

Ans. (1)

Sol. $y = xe^{x^2}$

$$\frac{dy}{dx} \Big|_{(1,e)} = (e \cdot e^{x^2} \cdot 2x + e^{x^2}) \Big|_{(1,e)} = 2 \cdot e + e = 3e$$

$$T : y - e = 3e(x - 1)$$

$$y = 3ex - 3e + e$$

$$y = (3e)x - 2e$$

$$\left(\frac{4}{3}, 2e\right) \text{ lies on it}$$

Option (1)

11. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$

$$-x + 4y + 7z = 0$$

$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is :

- (1) One (2) Three
 (3) Four (4) Two

Ans. (4)

Sol.
$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$$(8 - 7 \cos 2\theta) - 3(-2 - 7 \sin 3\theta) + 7(-\cos 2\theta - 4 \sin 3\theta) = 0$$

$$14 - 7 \cos 2\theta + 21 \sin 3\theta - 7 \cos 2\theta - 28 \sin 3\theta = 0$$

$$14 - 7 \sin 3\theta - 14 \cos 2\theta = 0$$

$$14 - 7(3 \sin \theta - 4 \sin^3 \theta) - 14(1 - 2 \sin^2 \theta) = 0$$

$$-21 \sin \theta + 28 \sin^3 \theta + 28 \sin^2 \theta = 0$$

$$7 \sin \theta [-3 + 4 \sin^2 \theta + 4 \sin \theta] = 0$$

$$\sin \theta, 4 \sin^2 \theta + 6 \sin \theta - 2 \sin \theta - 3 = 0$$

$$2 \sin \theta(2 \sin \theta + 3) - 1(2 \sin \theta + 3) = 0$$

$$\sin \theta = \frac{-3}{2}; \sin \theta = \frac{1}{2}$$

Hence, 2 solutions in $(0, \pi)$
Option (4)

12. If $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$, then $f'(1/2)$ is:

- (1) $\frac{6}{25}$ (2) $\frac{24}{25}$
(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

Ans. (2)

Sol.
$$\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt \quad f'\left(\frac{1}{2}\right) = ?$$

Differentiate w.r.t. 'x'
 $f(x) = 2x + 0 - x^2 f(x)$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1 + \frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

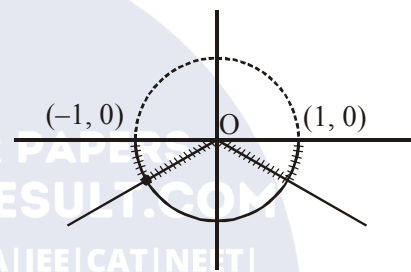
13. Let $f : (-1,1) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

- (1) Three elements (2) One element
(3) Five elements (4) Two elements

Ans. (1)

Sol. $f : (-1, 1) \rightarrow \mathbb{R}$

$$f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$$



Non-derivable at 3 points in $(-1, 1)$
Option (1)

14. Let $S = \left\{ (x,y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1 \right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$,

where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$,

where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$,

when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$,

when $r > 1$

Ans. (4)

Sol. $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

for $r > 1$, $\frac{y^2}{1+r} + \frac{x^2}{r-1} = 1$

$$e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)}$$

$$= \sqrt{\frac{(r+1) - (r-1)}{(r+1)}}$$

$$= \sqrt{\frac{2}{r+1}} = \sqrt{\frac{2}{r+1}}$$

Option (4)

15. If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K \left({}^{50}C_{25} \right)$, then K is equal to :
 (1) $2^{25} - 1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

Ans. (3)

Sol. $\sum_{r=0}^{25} {}^{50}C_r \cdot {}^{50-r}C_{25-r}$

$$= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \times \frac{(50-r)!}{(25)!(25-r)!}$$

$$= \sum_{r=0}^{25} \frac{50!}{25!25!} \times \frac{25!}{(25-r)!(r!)}$$

$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = (2^{25}) {}^{50}C_{25}$$

$\therefore K = 2^{25}$

Option (3)

16. Let N be the set of natural numbers and two functions f and g be defined as $f, g : N \rightarrow N$

such that : $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$

and $g(n) = n - (-1)^n$. The fog is :

- (1) Both one-one and onto
- (2) One-one but not onto
- (3) Neither one-one nor onto
- (4) onto but not one-one

Ans. (4)

Sol. $f(x) = \begin{cases} \frac{n+1}{2} & n \text{ is odd} \\ n/2 & n \text{ is even} \end{cases}$

$g(x) = n - (-1)^n \begin{cases} n+1 ; n \text{ is odd} \\ n-1 ; n \text{ is even} \end{cases}$

$f(g(n)) = \begin{cases} \frac{n}{2}; & n \text{ is even} \\ \frac{n+1}{2}; & n \text{ is odd} \end{cases}$

\therefore many one but onto

Option (4)

17. The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is :

- (1) 2
- (2) $\frac{4}{9}$
- (3) $\frac{15}{8}$
- (4) 1

Ans. (1)

Sol. $\alpha + \beta = \lambda - 3$

$\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$
 $= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$
 $= \lambda^2 - 4\lambda + 5$
 $= (\lambda - 2)^2 + 1$

$\therefore \lambda = 2$

Option (1)

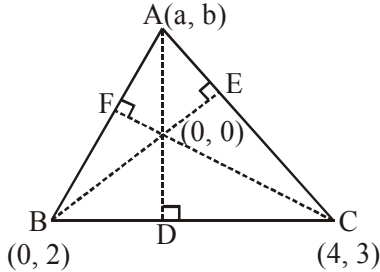
18. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

- (1) Fourth
- (2) Second
- (3) Third
- (4) First

Ans. (2)

Sol. $m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow b + 4a = 0 \dots\dots(i)$



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{b-2}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$

$\Rightarrow 3b - 6 = -4a \Rightarrow 4a + 3b = 6 \dots\dots(ii)$

From (i) and (ii)

$a = \frac{-3}{4}, b = 3$

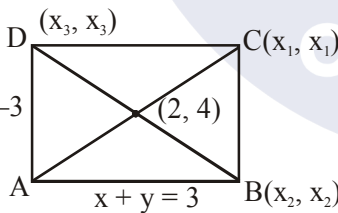
\therefore IInd quadrant.

Option (2)

19. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at $(2, 4)$, then one of its vertex is :

- (1) (2,6) (2) (2,1)
- (3) (3,5) (4) (3,6)

Ans. (4)



Sol. $x + y = -3$

Solving $x + y = 3$ and $x - y = -3$ $\Rightarrow A(0, 3)$

$\frac{x_1 + 0}{2} = 2; x_1 = 4$ similarly $y_1 = 5$

$C \Rightarrow (4, 5)$

Now equation of BC is $x - y = -1$ and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $x - y = -3$

Point D is $(3, 6)$

Option (4)

20. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors \vec{a} and \vec{b} are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

- (1) -3 (2) 4
- (3) 3 (4) -4

Ans. (4)

Sol. $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$

$\frac{\lambda - 2}{4\lambda - 2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\lambda = -4$

\therefore Option (4)

21. The value of $\cot\left(\sum_{n=1}^{19} \cot^{-1}\left(1 + \sum_{p=1}^n 2p\right)\right)$ is :

- (1) $\frac{22}{23}$ (2) $\frac{23}{22}$
- (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. (3)

Sol. $\cot\left(\sum_{n=1}^{19} \cot^{-1}(1 + n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19} \cot^{-1}(n^2 + n + 1)\right) = \cot\left(\sum_{n=1}^{19} \tan^{-1}\frac{1}{1 + n(n+1)}\right)$

$\sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1}n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A \cot \beta + 1}{\cot \beta - \cot A}$

$\frac{1\left(\frac{1}{20}\right) + 1}{1 - \frac{1}{20}} = \frac{21}{19}$

(Where $\tan A = 20, \tan B = 1$)

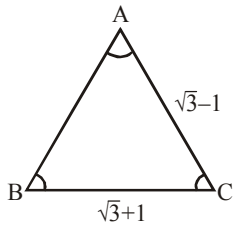
\therefore Option (3)

22. With the usual notation, in ΔABC , if $\angle A + \angle B = 120^\circ, a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$, then the ratio $\angle A : \angle B$, is :

- (1) 7 : 1 (2) 5 : 3
- (3) 9 : 7 (4) 3 : 1

Ans. (1)

Sol. $A + B = 120^\circ$



$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \left(\frac{C}{2} \right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})} \cot(30^\circ) = \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1$$

$$\frac{A-B}{2} = 45^\circ$$

$$\Rightarrow \begin{aligned} A-B &= 90^\circ \\ A+B &= 120^\circ \end{aligned}$$

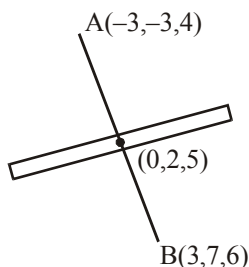
$$\begin{aligned} 2A &= 210^\circ \\ A &= 105^\circ \\ B &= 15^\circ \end{aligned}$$

\therefore Option (1)

23. The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angles, passes through which one of the following points ?

- (1) $(4, -1, 7)$ (2) $(4, 1, -2)$
 (3) $(-2, 3, 5)$ (4) $(2, 1, 3)$

Ans. (2)



$$\vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$$

$$\begin{aligned} p : 3(x - 0) + 5(y - 2) + 1(z - 5) &= 0 \\ 3x + 5y + z &= 15 \end{aligned}$$

\therefore Option (2)

24. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

- (1) $(P \wedge Q) \vee (\sim R)$
 (2) $(\sim P) \wedge (\sim Q \wedge R)$
 (3) $(\sim P) \vee (Q \wedge R)$
 (4) $P \vee (\sim Q \wedge R)$

Ans. (4)

Sol. It is obvious

\therefore Option (4)

25. On which of the following lines lies the point

of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$

and the plane, $x + y + z = 2$?

(1) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Ans. (3)

Sol. General point on the given line is

$$x = 2\lambda + 4$$

$$y = 2\lambda + 5$$

$$z = \lambda + 3$$

Solving with plane,

$$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$$

$$5\lambda + 12 = 2$$

$$5\lambda = -10$$

$$\boxed{\lambda = -2}$$

\therefore Option (3)

26. Let f be a differentiable function such that

$$f'(x) = 7 - \frac{3f(x)}{4x}, (x > 0) \text{ and } f(1) \neq 4.$$

Then $\lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right)$:

- (1) Exists and equals 4
- (2) Does not exist
- (3) Exist and equals 0
- (4) Exists and equals $\frac{4}{7}$

Ans. (1)

Sol. $f'(x) = 7 - \frac{3f(x)}{4x} \quad (x > 0)$

Given $f(1) \neq 4 \quad \lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right) = ?$

$$\frac{dy}{dx} + \frac{3y}{4x} = 7 \text{ (This is LDE)}$$

$$\text{IF} = e^{\int \frac{3}{4x} dx} = e^{\frac{3}{4} \ln|x|} = x^{\frac{3}{4}}$$

$$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx$$

$$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$f(x) = 4x + C \cdot x^{\frac{3}{4}}$$

$$f\left(\frac{1}{x}\right) = \frac{4}{x} + C \cdot x^{\frac{3}{4}}$$

$$\lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right) = \lim_{x \rightarrow 0^+} \left(4 + C \cdot x^{\frac{7}{4}}\right) = 4$$

∴ Option (1)

27. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the

point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter

when it is nearest to him. Then this nearest distance is :

- (1) $\frac{1}{2}$
- (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$
- (3) $\frac{1}{6}\sqrt{\frac{7}{3}}$
- (4) $\frac{\sqrt{5}}{6}$

Ans. (3)

Sol. $y - x^{3/2} = 7 \quad (x \geq 0)$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$$

$$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$$

$$\frac{3}{2} \cdot x^2 = \frac{1}{2} - x$$

$$3x^2 = 1 - 2x$$

$$3x^2 + 2x - 1 = 0$$

$$3x^2 + 3x - x - 1 = 0$$

$$(x + 1)(3x - 1) = 0$$

$$\therefore x = -1 \text{ (rejected)}$$

$$x = \frac{1}{3}$$

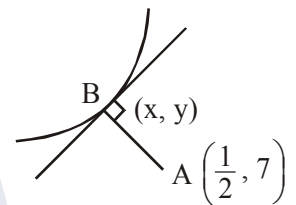
$$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$$

$$l_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$$

$$= \sqrt{\frac{3+4}{9 \times 12}}$$

$$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$$

Option (3)



28. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

- (1) $-4x^3 - 1$ (2) $4x^3 + 1$
 (3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Ans. (1)

Sol. $\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$

Put $x^3 = t$

$$3x^2 dx = dt$$

$$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$$

$$\frac{1}{3} \int t \cdot e^{-4t} dt$$

$$\frac{1}{3} \left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt \right]$$

$$-\frac{e^{-4t}}{48} [4t + 1] + c$$

$$-\frac{e^{-4x^3}}{48} [4x^3 + 1] + c$$

$$\therefore f(x) = -1 - 4x^3$$

Option (1)

(From the given options (1) is most suitable)

29. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy dy = 0$ which passes through (1,1) is :

- (1) A circle with centre on the y-axis
 (2) A circle with centre on the x-axis
 (3) An ellipse with major axis along the y-axis
 (4) A hyperbola with transverse axis along the x-axis

Ans. (2)

Sol. $(x^2 - y^2) dx + 2xy dy = 0$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

Solving we get,

$$\int \frac{2v}{v^2 + 1} dv = \int -\frac{dx}{x}$$

$$\ln(v^2 + 1) = -\ln x + C$$

$$(y^2 + x^2) = Cx$$

$$1 + 1 = C \Rightarrow C = 2$$

$$\boxed{y^2 + x^2 = 2x}$$

\therefore Option (2)

30. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20

(2) 25

(3) 13

(4) -25

Ans. (2)

Sol. $3\left(\frac{1}{2}r^2 \cdot \sin 120^\circ\right) = 27\sqrt{3}$

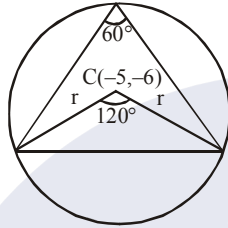
$$\frac{r^2 \sqrt{3}}{2 \cdot 2} = \frac{27\sqrt{3}}{3}$$

$$r^2 = \frac{108}{3} = 36$$

$$\text{Radius} = \sqrt{25 + 36 - C} = \sqrt{36}$$

$$\boxed{C = 25}$$

\therefore Option (2)



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