JEE MAINS 2018 15TH APRIL 2018 MORNING SHIFT
MATHEMATICS

61. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then:

(1) $P \subset Q$ and $Q - P \neq \phi$

(2) $Q \not\subset P$

(3) $P \not\subset Q$

(4) $P = Q$

62. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to:

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) 2

(4) $2\sqrt{2}$

63. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1+z+z^2+.....+z^{11}$ is equal to :

(1) $-1250\sqrt{3}i$

(2) $1250\sqrt{3}i$

(3) $1365\sqrt{3}i$

(4) $-1365\sqrt{3}i$

64. Let A be a 3×3 matrix such that $A^2-5A+7I=O$.

Statement –I: $A^{-1} = \frac{1}{7}(5I - A)$

Statement –II: The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Then

(1) Statement-I is true, but Statement-II is false.

(2) Statement-I is false, but Statement-II is true.

(3) Both the statements are true.

(4) Both the statements are false.

65. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is:

(1) 2014

(2) -175

(3) 2016

(4) -25

66. If $\frac{^{n+2}C_6}{^{n-2}C_6} = 11$, then n satisfies the equation:

(1) $n^2 + 3n - 108 = 0$

(2) $n^2 + 5n - 84 = 0$

(3) $n^2 + 2n - 80 = 0$

(4) $n^2 + n - 110 = 0$

67. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x > 0)$, are m and n respectively, then $\frac{m}{n}$ is equal to:

(1) 182

(2) $\frac{4}{5}$

(3) $\frac{5}{4}$

(4) 27

68. Let $a_1, a_2, a_3, ..., a_n, ...$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to :

(1) 306

(2) 153

(3) 612

(4) 204

69. The sum $\sum_{r=1}^{10}\left(r^2+1\right)\times(r!)$ is equal to:

(1) $(11)!$

(2) $10\times(11!)$

(3) $101\times(10!)$

(4) $11\times(11!)$

70. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is:

(1) -2

(2) $-\frac{1}{2}$

(3) $\frac{1}{2}$

(4) 2

71. Let $a, b \in R, (a \neq 0)$. If the function f defined as

$$f(x)=\begin{cases} \frac{2x^2}{a} & ,0 \leq x \leq 1 \\ a & ,1 \leq x < \sqrt{2} \\ \frac{2b^2-4b}{x^3} & ,\sqrt{2} \leq x < \infty \end{cases}$$

Is continuous in the interval [0,∞), then an ordered pir (*a*,*b*) is:

(1) $\left(\sqrt{2}, 1-\sqrt{3}\right)$

(2) $\left(-\sqrt{2}, 1+\sqrt{3}\right)$

(3) $\left(\sqrt{2}, -1+\sqrt{3}\right)$

(4) $\left(-\sqrt{2}, 1-\sqrt{3}\right)$

72. Let $f(x)=\sin^4 x+\cos^4 x$. Then *f* is an increasing function in the interval :

(1) $\left]0, \frac{\pi}{4}\right[$

(2) $\left]\frac{\pi}{4},\frac{\pi}{2}\right[$

(3) $\left]\frac{\pi}{2},\frac{5\pi}{8}\right[$

(4) $\left]\frac{5\pi}{8},\frac{3\pi}{4}\right[$

73. Let C be a curve given by $y(x)=1+\sqrt{4x-3}, x>\frac{3}{4}$. If P is a point on C, such that the tangent at P has lope $\frac{2}{3}$, then a point through which the normal at P passes, is:

(1) (2,3)

(2) (4,-3)

(3) (1,7)

(4) (3,-4)

74. The inegeral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to:

(where C is a constant of integration.)

(1) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

(2) $-2\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(3) $-\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(4) $2\sqrt{\dfrac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

75. The value of the integral $\displaystyle\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where $[x]$ denotes the greatest integer less than or equal to x, is :

(1) 6

(2) 3

(3) 7

(4) $\dfrac{1}{3}$

76. For $x \in R,\ x \neq 0$, if $y(x)$ is a differentiable function such that $x\displaystyle\int_1^x y(t)dt = (x+1)\int_1^x ty(t)dt$, then $y(x)$ equals:

(where C is a constant.)

(1) $\frac{C}{x}e^{-\frac{1}{x}}$

(2) $\frac{C}{x^2}e^{-\frac{1}{x}}$

(3) $\frac{C}{x^3}e^{-\frac{1}{x}}$

(4) $Cx^3e^{-\frac{1}{x}}$

77. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$,

where $0 \le x \le \frac{\pi}{2}$, and $y(0)=1$, is given by:

(1) $y=1-\frac{x}{\sec x+\tan x}$

(2) $y^2=1+\frac{x}{\sec x+\tan x}$

(3) $y^2=1-\frac{x}{\sec x+\tan x}$

(4) $y=1+\frac{x}{\sec x+\tan x}$

78. A ray of light is incident along a line which meets another line, $7x-y+1=0$, at the point (0, 1). The ray is then reflected from this point along the line, $y+2x=1$. Then the equation of the line of incidence of the ray of light is :

(1) $41x-38y+38=0$

(2) $41x-25y+25=0$

(3) $41x+38y-38=0$

(4) $41x-25y+25=0$

79. A straight line through origin O meets the lines $3y=10-4x$ and $8x+6y+5=0$ at points A and B respectively. Then O divides the segment AB in the ratio :

(1) 2:3

(2) 1:2

(3) 4:1

(4) 3:4

80. Equation of the tangent to the circle, at the point (1, −1), whose centre is the point of intersection of the straight lines $x-y=1$ and $2x+y=3$ is :

(1) $4x+y-3=0$

(2) $x + 4y+3=0$

(3) $3x- y-4=0$

(4) $x- 3y -4=0$

81. P and Q are two distinct points on the parabola, $y^2=4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is :

(1) 2

(2) 4

(3) 6

(4) 8

82. A hyperbola whose transverse axis is along the major axis of the conic, $2\left(\frac{x^2}{3}+\frac{y^2}{4}\right)=4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ?

(1) (0,2)

(2) $\left(\sqrt{5},2\sqrt{2}\right)$

(3) $\left(\sqrt{10},2\sqrt{3}\right)$

(4) $\left(5, 2\sqrt{3}\right)$

83. ABC is a triangle in a plane with vertices A(2, 3, 5), B(−1, 3, 2) and C(λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of ($\lambda^3+\mu^3+5$) is :

(1) 1130

(2) 1348

(3) 676

(4) 1077

84. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is:

(1) 4

(2) 1

(3) 2

(4) 3

85. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is :

(1) $\vec{a}+\vec{b}+\vec{c}$

(2) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

(3) $\vec{0}$

(4) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

86. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6 ; then the mean deviation from the mean of the data is :

(1) 2.4

(2) 2.8

(3) 2.5

(4) 2.6

87. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is :

(1) $\frac{240}{729}$

(2) $\frac{192}{729}$

(3) $\frac{256}{729}$

(4) $\frac{496}{729}$

88. If A>0, B>0 and $A+B=\frac{\pi}{6}$, then the minimum value of tanA+tanB is :

(1) $\sqrt{3}-\sqrt{2}$

(2) $2-\sqrt{3}$

(3) $4-2\sqrt{3}$

(4) $\frac{2}{\sqrt{3}}$

89. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is :

(1) $36\sqrt{3}$

(2) 54

(3) $54\sqrt{3}$

(4) 108

90. The contrapositive of the following statement,

"If the side of a square doubles, then its area increases four times", is:

(1) If the side of a square is not doubled, then its area does not increase four times.

(2) If the area of a square increases four times, then its side is doubled.

(3) If the area of a square increases four times, then its side is not doubled.

(4) If the area of a square does not increase four times, then its side is not doubled.

Part-2

61. Solve for the set P as follows,

$$\rho(r) \propto \frac{1}{r}$$

Solve for the set Q as follows,

$$\frac{32}{23}\,\mu F$$

Therefore, both the sets are equal to each other.

62. The given equation is as follows,

$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

Rewrite the above equation and square both the sides.

$400\ \Omega$

Solve for $P_2 > P_1 > P_3$ at $7\ \Omega$ and $45°$.

$\frac{2E_0}{c}\hat{j}\cos kz \cos \omega t$

63. The value of 27.5 cm is calculated as,

9 mm

It is given that the variable 10^{20} is a real number. Thus, the imaginary part of $\frac{h^2}{4\pi m^2 r^3}$ will be zero.

4×10^{-2} gm

The complex number is written as follows,

6.9 mA

The sum of given geometric series is,

$$\lambda, P_{eff} = K\left(\frac{1}{\lambda}\right)^2$$

$$\frac{20}{3}\,\Omega$$

Further, substitute the above expression,

$0.1\,\text{cm}$

$$P = a^{1/2} b^2 c^3 d^{-4}$$

Therefore, the required value is,

$$\frac{\Delta P}{P} = \left[\left(\frac{1}{2}\frac{\Delta a}{a}\right) + \left(2\frac{\Delta b}{b}\right) + \left(3\frac{\Delta c}{c}\right) + \left(4\frac{\Delta d}{d}\right)\right] \times 100\,\%$$

64. The given function is as follows.

$$\begin{aligned}\frac{\Delta P}{P} &= \left[\left(\frac{1}{2}\times 2\right) + (2\times 1) + (3\times 3) + (4\times 5)\right]\% \\ &= [1 + 2 + 9 + 20]\,\% \\ &= 32\,\%\end{aligned}$$

Solve the given equation.

$32\,\%$

Therefore, statement-I is true.

Solve the equation in statement-II,

$$s = ut + \frac{1}{2}at^2$$

Further solve the above equation.

$$s = (0)t + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}}$$

Therefore, statement-II is also true.

65. The given matrix is as follows,

$$(d + 200)$$

The determinant of given expression is calculated as,

$$t = \sqrt{\frac{2d}{2}} \qquad \ldots\ldots (1)$$

The square of matrix A is calculated as,

$(d + 200)$

$$\sqrt{\frac{2d}{2}} = \sqrt{\frac{2(d+200)}{4}}$$

Substitute $t = \sqrt{\frac{2(d+200)}{4}}$ for $d = \frac{(d+200)}{2}$ and

$d = 200\text{ m}$

$t = \sqrt{\frac{2(200\text{ m})}{2}}$ for A in equation (1).

$= 10\sqrt{2}\text{ s}$

$10\sqrt{2}\text{ s}$

66. The given expression is as follows,

M'

Solve the given expression.

υ'

Further solve the above equation.

$$2M'\upsilon'\sin\theta = M\upsilon\cos45^\circ + M\upsilon\cos30^\circ$$

$$2M'\upsilon'\sin\theta = \frac{M\upsilon}{\sqrt{2}} + \frac{\sqrt{3}M\upsilon}{2}$$

$$2M'\upsilon'\sin\theta = M\upsilon\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)$$

Among the options, *n* satisfies the equation

$$2M'\upsilon'\cos\theta = -M\upsilon\sin45^\circ + M\upsilon\sin30^\circ$$

$$2M'\upsilon'\cos\theta = -\frac{M\upsilon}{\sqrt{2}} + \frac{M\upsilon}{2}$$

$$2M'\upsilon'\cos\theta = M\upsilon\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right).$$

$$\frac{2M'\upsilon'\sin\theta}{2M'\upsilon'\cos\theta} = \frac{M\upsilon\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)}{M\upsilon\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}$$

67. The expansion of the expression s $\tan\theta = \frac{\left(\frac{\sqrt{2}+\sqrt{3}}{2}\right)}{\left(\frac{1-\sqrt{2}}{2}\right)}$

$$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$$

is written as follows.

θ

The value of *r* is calculated for the coefficient of

$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$.

ΔPQR

The value of r is calculated for the coefficient of

$$h=\sqrt{1^2-\left(\frac{x}{2}\right)^2}$$

$$=\frac{1}{2}\sqrt{4-x^2}$$

.

$$v=\frac{dh}{dt}$$

The coefficient of $\frac{dh}{dt}$ and

$$\frac{dh}{dt}=\frac{d}{dt}\left(\frac{1}{2}\sqrt{4-x^2}\right)$$

$$=\frac{1}{2}\frac{d}{dx}\left(\frac{1}{2}\sqrt{4-x^2}\right)\frac{dx}{dt}$$

$$=\frac{1}{4}\left(\frac{1}{\sqrt{4-x^2}}\right)(-2x)\frac{dx}{dt}$$

$$=-\frac{x}{2\sqrt{4-x^2}}\frac{dx}{dt}$$

are m and n respectively. So, the ratio is,

$$\frac{dh}{dt}=-\frac{1}{2\sqrt{\frac{4}{x^2}-1}}\frac{dx}{dt}$$

68. The given series is as follows,

$$\sqrt{\frac{4}{x^2}-1}$$

The numbers $\sqrt{\frac{4}{x^2}-1}$ are the terms in an A.P. Thus,

$$\frac{dh}{dt}$$

The sum of first 17 terms of A.P. series is calculated as,

$$T\cos\theta = mg$$

69. The sum of expression is calculated as,

$$T\sin\theta = \frac{mv^2}{r}$$

Further solve the above equation.

$$\tan\theta = \frac{v^2}{rg}$$

70. The given expression with the limits is calculated as follows.

$$\tan 45° = \frac{v^2}{(0.4\text{ m})(10\text{ ms}^{-2})}$$

$$v^2 = 4\text{ m}^2\text{s}^{-2}$$

$$v = \sqrt{4\text{ m}^2\text{s}^{-2}}$$

$$v = 2\text{ ms}^{-1}$$

71. For the interval 2 ms^{-1}, the function $I_{disc} = \frac{MR^2}{2}$ is continuous.

Thus, the function is also continuous at

$$I_{removed} = \frac{1}{2}\left(\frac{M}{16}\right)\left(\frac{R^2}{16}\right) + \left(\frac{M}{16}\right)\left(\frac{9R^2}{16}\right)$$

$$= \frac{MR^2 + 18MR^2}{512} \text{ and}$$

$$= \frac{19MR^2}{512}$$

$$I_{remaining} = \frac{MR^2}{2} - \frac{19MR^2}{512}.$$

$$= \frac{237MR^2}{512}$$

The limit of the function at $\frac{237MR^2}{512}$ is calculated as,

$$\rho = \frac{m}{v} = \frac{k}{r}$$

The limit of the function at $m = \frac{kv}{r}$ is calculated as,

$$g_{inside} = \frac{Gmr}{R^3}$$
$$= \left(\frac{Gr}{R^3}\right)\left(\frac{kv}{r}\right)$$
$$= \frac{Gkv}{R^3}$$

For $\frac{Gkv}{R^3}$, the value of *b* from above equation is calculated as,

$$g_{out} = \frac{Gm}{r^2}$$

For $F = Y\alpha_L A\Delta t$, the value of *b* is calculated as,

$$F = \left(2\times10^{11}\ \text{Nm}^{-2}\right)\left(1.2\times10^{-5}\ \text{K}^{-1}\right)\left(40\times10^{-4}\ \text{m}^2\right)(10)$$
$$= 9.6\times10^4\ \text{N}$$
$$= 1\times10^5\ \text{N}$$

The value of *b* comes out to be an imaginary number which is not possible.

Therefore, the ordered pair $Q=\frac{\pi r^4}{8\eta}\frac{\Delta P}{L}$ is
$$\frac{P_1 r_1^4}{l_1}=\frac{P_2 r_2^4}{l_2}$$
$$\frac{P_1 r_1^4}{l_1}=\frac{4P_1 r_2^4}{\frac{I_1}{4}}$$
$$r_2^4=\frac{r_1^4}{16}$$
$$r_2=\frac{r_1}{2}$$
.

72. The given function is written as follows,

$$u_{\text{initial}}=\frac{5}{2}NRT$$

Differentiate the function
$$u_{\text{final}}=\frac{3}{2}(2nRT)+\frac{5}{2}(N\text{-}n)RT$$
$$=\frac{1}{2}nRT+\frac{5}{2}NRT$$
with respect to *x*.

$$U_{total} = \frac{1}{2}nRT + \frac{5}{2}NRT - \frac{5}{2}NRT$$

$$= \frac{1}{2}nRT$$

Determine the interval for which the above function is increasing,

$$a = a_0 e^{\frac{-bt}{m}}$$

73. Let point $\begin{matrix} E \propto a^2 \\ a \propto E \end{matrix}$ be a point that lies on the curve C.

Differentiate the given equation with respect to *x*.

$$a = \frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

$$\frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

$$= \frac{10^{-2}t}{0.1}$$

$$= \frac{t}{10}$$

$$\frac{a_0}{\sqrt{2}} = a_0 e^{-\frac{t}{10}}$$

Thus, the point P is obtained as $\frac{1}{\sqrt{2}} = e^{-\frac{t}{10}}$.

$$\ln\sqrt{2} = \frac{t}{10}$$

$$t = 3.5 \text{ sec}$$

The slope of normal at point P $v = \frac{\omega}{k}$ is equal to $v = \frac{200\pi}{\left(\frac{5\pi}{4}\right)}$.

$$= 160 \text{ m/s}$$

The equation of normal is calculated as follows,

$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$

Among all the given options, it is clear that normal passes through the point $C = \frac{\varepsilon_0 A}{3}$.

74. Integrate the given expression.

$$C=\frac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{\frac{k\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$$

$$\frac{\varepsilon_0 A}{3}=\frac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{\frac{k\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$$

Substitute $3k = 2.4k + 3$ for $\sigma_1 = \varepsilon_0 \upsilon B, \sigma_2 = -\varepsilon_0 \upsilon B$ and

$k = 5$

then differentiate of this expression.

$$\frac{(A - C)}{D}$$

Substitute these values in the given expression and then integrate.

$MnR^2 t$

Further solve the above expression.

-4500 J …… (1)

The value of cosine in terms of *x* is calculated as, 1.37

Substitute this value in equation (1).

$$F_G = \frac{GMm}{(R+h)^2}$$

75. The given integral is as follows,

$$(4\pi\mu Bb)\Delta n \qquad \ldots\ldots (1)$$

Rewrite the equation (1) by using the following property,

1.67×10^5 J

Thus, equation (1) becomes,

1.9 Hz …… (2)

Add equation (1) and equation (2).

170 Hz

76. The given integral is written as follows,

$$\rho(r) \propto \frac{1}{r}$$

Differentiate above equation with respect to *x*.

$$\frac{32}{23}\,\mu F$$

Differentiate the above equation.

$$\sigma_1 = \varepsilon_0 \upsilon B, \sigma_2 = -\varepsilon_0 \upsilon B$$

Further solve the above differential equation.

$$400\,\Omega$$

77. The given differential equation is written as follows,

$$P_2 > P_1 > P_3 \qquad \ldots\ldots (1)$$

Substitute $7\,\Omega$ and 45° for $\frac{2E_0}{c}\hat{j}\cos kz \cos \omega t$.

Differentiate 27.5 cm with respect to *x*,

$$9\text{ mm}$$

Substitute the values in equation (1).

10^{20} ….. (2)

The integrated factor of the above equation is,

$\frac{h^2}{4\pi m^2 r^3}$

The solution of differential equation (2) is calculated as follows,

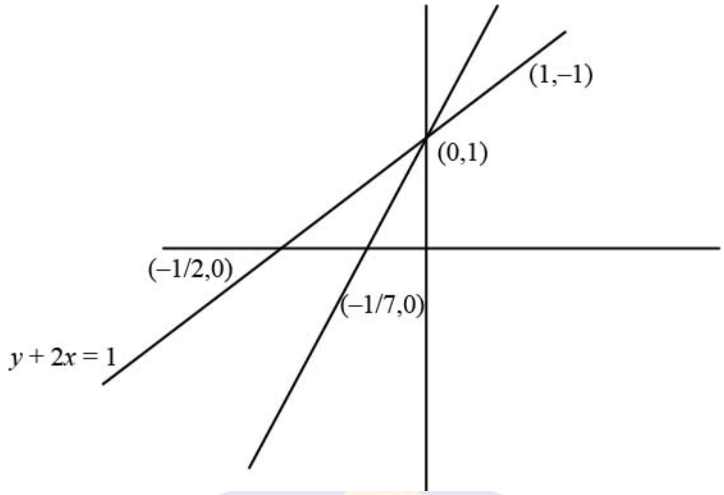
4×10^{-2} gm

78. The equation of incident line is as follows,

6.9 mA …… (1)

Let a point $\lambda, P_{eff} = K\left(\frac{1}{\lambda}\right)^2$ be on the line $\frac{20}{3}\Omega$ and the image of $0.1\,\text{cm}$ lie on the incidence line in $P = a^{1/2}b^2c^3d^{-4}$.

The following figure shows the incident line.



The equation of line is given by,

$$\frac{\Delta P}{P}=\left[\left(\frac{1}{2}\frac{\Delta a}{a}\right)+\left(2\frac{\Delta b}{b}\right)+\left(3\frac{\Delta c}{c}\right)+\left(4\frac{\Delta d}{d}\right)\right]\times 100\,\%$$

From the above equation, the value of *x* is calculated as,

$$\frac{\Delta P}{P}=\left[\left(\frac{1}{2}\times 2\right)+(2\times 1)+(3\times 3)+(4\times 5)\right]\%$$
$$=[1+2+9+20]\,\%$$
$$=32\,\%$$

The value of *y* is calculated as,

32 %

Substitute the values of *x* and *y* in equation (1).

$$s = ut + \frac{1}{2}at^2$$

$$s = (0)t + \frac{1}{2}at^2$$

Substitute $s = \frac{1}{2}at^2$ for $(d + 200)$ in equation (1).

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{2d}{2}}$$

79. Let the straight line through origin $(d + 200)$ be given by,

$$t = \sqrt{\frac{2(d + 200)}{4}}$$

$$\sqrt{\frac{2d}{2}} = \sqrt{\frac{2(d + 200)}{4}}$$

The above line intersects the line $d = \frac{(d + 200)}{2}$ at point

$$d = 200 \text{ m}$$

A, then,

$$t = \sqrt{\frac{2(200\ \text{m})}{2}} \qquad \ldots\ldots (1)$$

$$= 10\sqrt{2}\ \text{s}$$

Again the line through the origin meets line $10\sqrt{2}\ \text{s}$ at point B thus,

M' …… (2)

Divide equation (1) by equation (2),

υ'

80. The equation of the first straight line is,

$$2M'\upsilon'\sin\theta = M\upsilon\cos45° + M\upsilon\cos30°$$

$$2M'\upsilon'\sin\theta = \frac{M\upsilon}{\sqrt{2}} + \frac{\sqrt{3}M\upsilon}{2}$$

$$2M'\upsilon'\sin\theta = M\upsilon\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)$$

…… (1)

The equation of the second straight line is,

$$2M'\upsilon'\cos\theta = -M\upsilon\sin 45° + M\upsilon\sin 30°$$

$$2M'\upsilon'\cos\theta = -\frac{M\upsilon}{\sqrt{2}} + \frac{M\upsilon}{2}$$

$$2M'\upsilon'\cos\theta = M\upsilon\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

…… (2)

Add both equation (1) and (2) to obtain the value of *x* as follows.

$$\frac{2M'\upsilon'\sin\theta}{2M'\upsilon'\cos\theta} = \frac{M\upsilon\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)}{M\upsilon\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}$$

$$\tan\theta = \frac{\left(\frac{\sqrt{2}+\sqrt{3}}{2}\right)}{\left(\frac{1-\sqrt{2}}{2}\right)}$$

$$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$$

Substitute θ for *x* in equation (1).

$$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$$

The centre of circle is the point where the straight lines ΔPQR

and $h=\sqrt{1^2-\left(\frac{x}{2}\right)^2}$ intersect. Thus, the centre of circle is at

$$=\frac{1}{2}\sqrt{4-x^2}$$

$$v=\frac{dh}{dt}.$$

The equation of the circle is calculated as,

$$\frac{dh}{dt}$$

The equation of tangent to the circle at point

$$\frac{dh}{dt}=\frac{d}{dt}\left(\frac{1}{2}\sqrt{4-x^2}\right)$$

$$=\frac{1}{2}\frac{d}{dx}\left(\frac{1}{2}\sqrt{4-x^2}\right)\frac{dx}{dt}$$

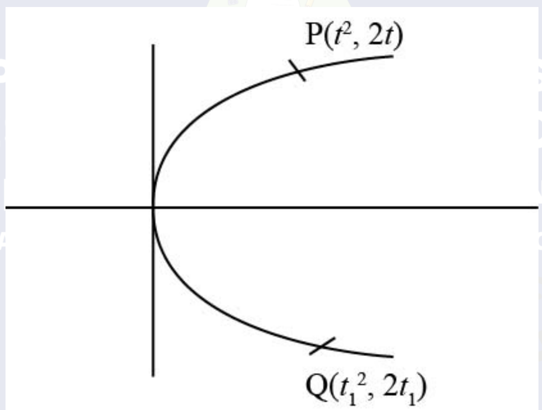
is calculated as,

$$=\frac{1}{4}\left(\frac{1}{\sqrt{4-x^2}}\right)(-2x)\frac{dx}{dt}$$

$$=-\frac{x}{2\sqrt{4-x^2}}\frac{dx}{dt}$$

$$\frac{dh}{dt} = -\frac{1}{2\sqrt{\frac{4}{x^2}-1}}\frac{dx}{dt}$$

81. Consider point P as $\sqrt{\frac{4}{x^2}-1}$ and consider point Q as $\sqrt{\frac{4}{x^2}-1}$ on the parabola, $\frac{dh}{dt}$.



The normal at point P $T\cos\theta = mg$ passes through point Q

$T\sin\theta = \frac{mv^2}{r}$ and the equation is given as,

$$\tan\theta = \frac{v^2}{rg} \qquad \ldots\ldots (1)$$

Differentiate the above equation with respect to *t*.

$$\tan 45^\circ = \frac{v^2}{(0.4\ \text{m})(10\ \text{ms}^{-2})}$$

$$v^2 = 4\ \text{m}^2\text{s}^{-2}$$

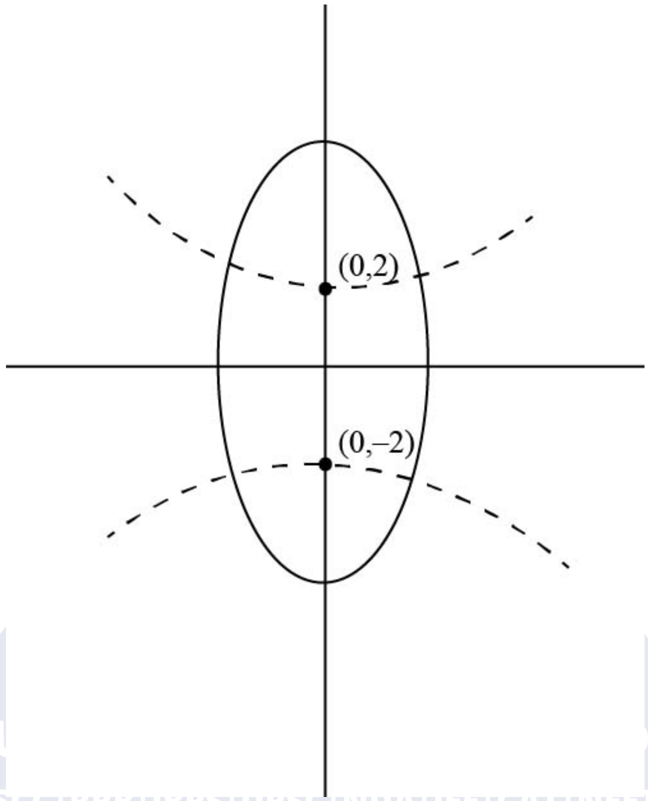
$$v = \sqrt{4\ \text{m}^2\text{s}^{-2}}$$

$$v = 2\ \text{ms}^{-1}$$

Square both sides of equation (1).

$2\ \text{ms}^{-1}$

82. The following figure represents the ellipse as per given the point of the hyperbola $I_{disc} = \frac{MR^2}{2}$.



For ellipse,

The equation of ellipse is written as,

$$I_{removed} = \frac{1}{2}\left(\frac{M}{16}\right)\left(\frac{R^2}{16}\right) + \left(\frac{M}{16}\right)\left(\frac{9R^2}{16}\right)$$

$$= \frac{MR^2 + 18MR^2}{512}$$

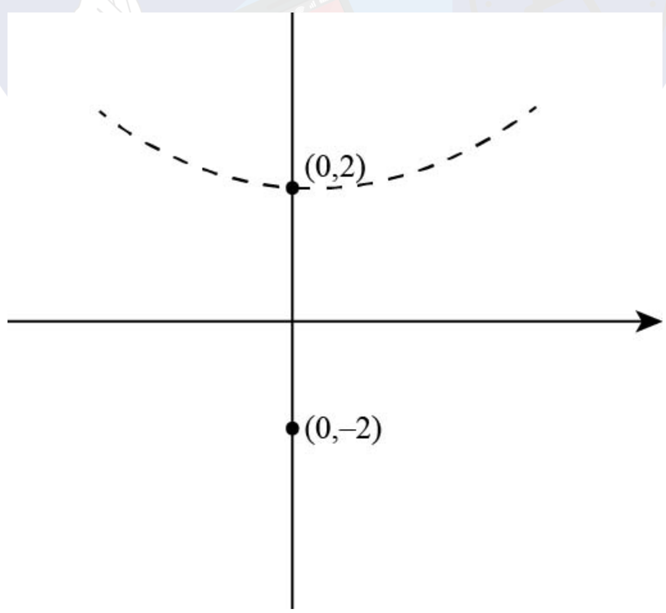
$$= \frac{19MR^2}{512}$$

Let the foci of ellipse be $I_{remaining} = \frac{MR^2}{2} - \frac{19MR^2}{512} = \frac{237MR^2}{512}$. Thus, the eccentricity of ellipse is calculated as,

$$\frac{237MR^2}{512}$$

Thus, the point $\rho = \frac{m}{v} = \frac{k}{r}$ or $m = \frac{kv}{r}$ does not lie on the parabola.

For hyperbola,



Equation of the hyperbola is given by,

$$g_{inside} = \frac{Gmr}{R^3}$$
$$= \left(\frac{Gr}{R^3}\right)\left(\frac{kv}{r}\right)$$
$$= \frac{Gkv}{R^3}$$

The eccentricity of the hyperbola is calculated as,

$$\frac{Gkv}{R^3}$$

Given that $g_{out} = \frac{Gm}{r^2}$, therefore,

$$F = Y\alpha_L A\Delta t$$

Thus, the equation of the ellipse is,

$$F = \left(2\times10^{11}\ Nm^{-2}\right)\left(1.2\times10^{-5}\ K^{-1}\right)\left(40\times10^{-4}\ m^2\right)(10)$$
$$= 9.6\times10^{4}\ N$$
$$= 1\times10^{5}\ N$$

All the options are checked by substituting the given points on the hyperbola, Check option (4).

For the point $Q = \frac{\pi r^4}{8\eta}\frac{\Delta P}{L}$,

$$\frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

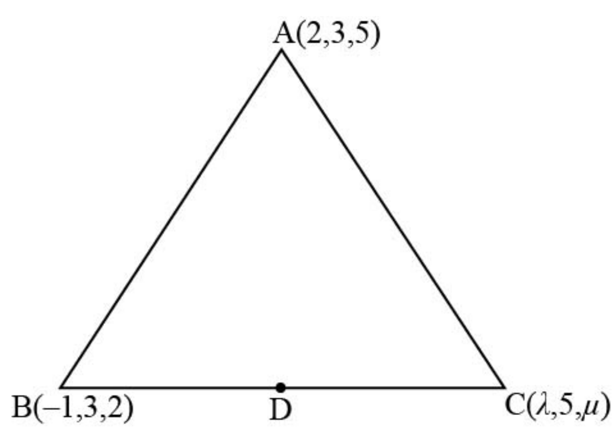
$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{\frac{l_1}{4}}$$

$$r_2^4 = \frac{r_1^4}{16}$$

$$r_2 = \frac{r_1}{2}$$

Therefore, option (4) is correct.

83. The following figure shows the triangle ABC with all its vertices.



The coordinates of point D is as follows,

$$u_{\text{initial}} = \frac{5}{2}\text{NRT}$$

The direction cosine of median AD is given by,

$$u_{\text{final}} = \frac{3}{2}(2\text{nRT}) + \frac{5}{2}(\text{N-n})\text{RT}$$
$$= \frac{1}{2}\text{nRT} + \frac{5}{2}\text{NRT}$$

The vector AD is written as follows,

$$U_{\text{total}} = \frac{1}{2}\text{nRT} + \frac{5}{2}\text{NRT} - \frac{5}{2}\text{NRT}$$
$$= \frac{1}{2}\text{nRT}$$

From the above expression,

$$a = a_0 e^{\frac{-bt}{m}}$$

From the above expression, the value of $\begin{matrix} E \propto a^2 \\ a \propto E \end{matrix}$ is found out to

$$a = \frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

be 7 and the value of $\frac{a_0}{\sqrt{2}} = \frac{bt}{m}$ is found out to be 10.

$$= \frac{10^{-2} t}{0.1}$$

$$= \frac{t}{10}$$

Thus, the value of given expression is calculated as,

$$\frac{a_0}{\sqrt{2}} = a_0 e^{-\frac{t}{10}}$$

$$\frac{1}{\sqrt{2}} = e^{-\frac{t}{10}}$$

$$\ln\sqrt{2} = \frac{t}{10}$$

$$t = 3.5 \text{ sec}$$

84. The given equation of line is as follows,

$$v = \frac{\omega}{k}$$

$$v = \frac{200\pi}{\left(\frac{5\pi}{4}\right)}$$

$$= 160 \text{ m/s}$$

The given two lines are coplanar. Thus,

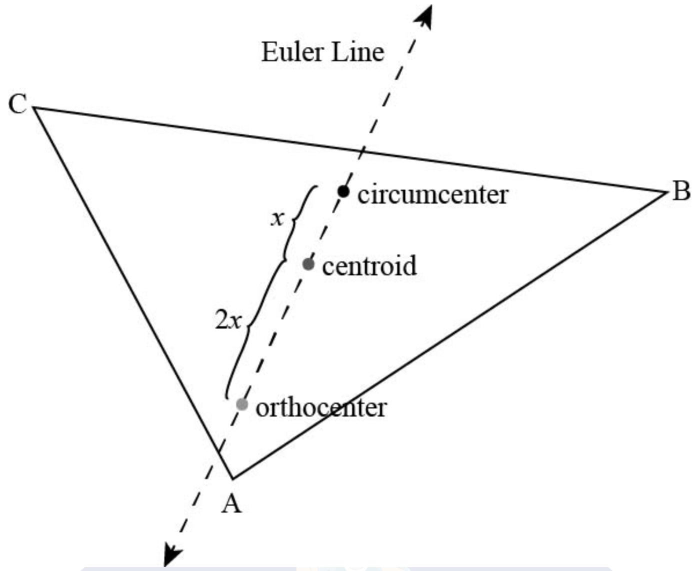
$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

Therefore, there are three possible values of λ.

85. The expression for the centroid of the triangle ABC for the given position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is as follows,

$$\text{Centroid} \equiv \left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$$

The relationship between the centroid, circumcenter and orthocenter is shown in the following diagram.



Therefore,

$$\text{Orthocentre} = 3(\text{centroid}) - 2(\text{circumcenter})$$

$$\text{Orthocentre} = 3\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) - 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right)$$

$$= \left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$$

86. The mean of 5 observations is 5. This can be expressed in the numerical form as,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$5 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 25$$

$$\sum_{i=1}^{5} x_i = 25 \qquad \ldots\ldots (1)$$

The variance of the observations is calculated as,

$$\sigma^2 = 124$$

$$\frac{\sum x_1^2}{5} - \left(\bar{x}\right)^2 = 124$$

$$\sum x_1^2 = 745$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 745$$

Substitute the given three observations,

Consider as the three observations x_3, x_4 and x_5 are 1, 2 and 6 respectively.

Therefore, from the above equation,

$$x_1^2 + x_2^2 + \left(1^2\right) + 2^2 + 6^2 = 745$$

$$x_1^2 + x_2^2 = 704 \qquad \ldots\ldots (2)$$

Similarly,

From equation (1),

$$x_1 + x_2 + 1 + 2 + 6 = 25$$

$$x_1 + x_2 = 16 \qquad \ldots\ldots (3)$$

From equation (2) and equation (3),

$$(x_1 + x_2)^2 = (16)^2$$

$$2x_1x_2 + 704 = 256$$

$$x_1x_2 = \frac{256 - 704}{2}$$

$$x_1x_2 = -224$$

The mean deviation is calculated as follows,

$$\frac{\sum |x_1 - 5|}{5} = \frac{|x_1 - 5| + |x_2 - 5| + |1 - 5| + |2 - 5| + |6 - 5|}{5}$$

$$= \frac{8 + |x_1 - 5| + |16 - x_1 - 5|}{5}$$

$$= \frac{8 + 6}{5}$$

$$= 2.8$$

87. The experiment succeeds twice as often as it fails. The condition is mathematically written as,

$p = 2q$

The total number of experiments is given by,

$p + q = 1$

From above equations, the value of q is $\frac{1}{3}$ and the value of p is $\frac{2}{3}$.

The probability of at least 5 successes in the six trails is given by,

$$^{6}C_5 p^5 q + {}^{6}C_6 p^6 = 6\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + 1\left(\frac{2}{3}\right)^6$$

$$= \frac{256}{729}$$

88. Consider the given expression as,

$y = \tan A + \tan B$

Differentiate the above equation with respect to A.

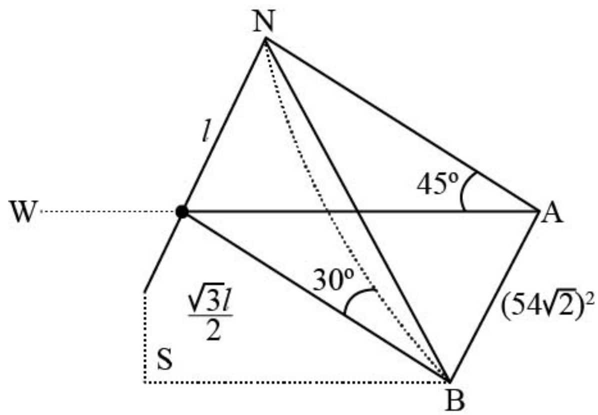
$$\frac{dy}{dA} = \sec^2 A - \sec^2 B$$
$$= \sec^2 A - \sec^2\left(\frac{\pi}{6} - A\right) \quad \left(\because A + B = \frac{\pi}{6}\right)$$

The expression $(\tan A + \tan B)$ increases in the interval $\left[\frac{\pi}{12}, \frac{\pi}{6}\right]$ and decreases in the interval $\left[0, \frac{\pi}{12}\right]$.Thus, it is can be concluded that the expression $(\tan A + \tan B)$ is minimum when A and B is equal to $\frac{\pi}{12}$.

Therefore, the minimum value of the expression is calculated as,

$$y_{min} = 2\tan\frac{\pi}{12}$$
$$= \left(2 - \sqrt{3}\right)2$$
$$= 4 - 2\sqrt{3}$$

89. The following figure is a pictorial representation of the given conditions.



From the figure the height of the tower is calculated as,

$$\frac{l^2}{4}+\left(54\sqrt{2}\right)^2=\frac{3l^2}{4}$$

$$\left(54\right)^2\times 2\times 2=l^2$$

$$l=54\times 2$$

$$=108\text{ m}$$

90. Assume the side of a square doubles to be represented as p. The area of square increases by four times. Let it be represents by q. Thus, the contra positive of $p\rightarrow q$ is $\sim q\rightarrow \sim p$.

Therefore, the correct option is (4).