

IIT JEE MAINS 2016 10TH APRIL 2016
MATHEMATICS

61. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and

$Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then:

- (1) $P \subset Q$ and $Q - P \neq \phi$
- (2) $Q \not\subset P$
- (3) $P \not\subset Q$
- (4) $P = Q$

62. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$,

then $\sqrt{4x^2 - 1}$ is equal to:

- (1) $\frac{3}{4}$
- (2) $\frac{1}{2}$
- (3) 2
- (4) $2\sqrt{2}$

63. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1 + z + z^2 + \dots + z^{11}$ is equal to :

- (1) $-1250\sqrt{3}i$
- (2) $1250\sqrt{3}i$
- (3) $1365\sqrt{3}i$
- (4) $-1365\sqrt{3}i$

64. Let A be a 3×3 matrix such that $A^2 - 5A + 7I = O$.

Statement -I: $A^{-1} = \frac{1}{7}(5I - A)$

Statement -II: The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Then

- (1) Statement-I is true, but Statement-II is false.
- (2) Statement-I is false, but Statement-II is true.
- (3) Both the statements are true.
- (4) Both the statements are false.

65. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix

$(A^{2016} - 2A^{2015} - A^{2014})$ is:

(1) 2014

(2) -175

(3) 2016

(4) -25

66. If $\frac{{}^{n+2}C_6}{{}^{n-2}C_6} = 11$, then n satisfies the equation:

(1) $n^2 + 3n - 108 = 0$

(2) $n^2 + 5n - 84 = 0$

(3) $n^2 + 2n - 80 = 0$

(4) $n^2 + n - 110 = 0$

67. If the coefficients of x^{-2} and x^{-4} in the expansion of

$\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, ($x > 0$), are m and n respectively, then $\frac{m}{n}$ is equal

to:

(1) 182

(2) $\frac{4}{5}$

(3) $\frac{5}{4}$

(4) 27

68. Let $a_1, a_2, a_3, \dots, a_n, \dots$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to :

(1) 306

(2) 153

(3) 612

(4) 204

69. The sum $\sum_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to:

(1) $(11)!$

(2) $10 \times (11!)$

(3) $101 \times (10!)$

(4) $11 \times (11!)$

70. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)^2}{2x \tan x - x \tan 2x}$ is:

(1) -2

(2) $-\frac{1}{2}$

(3) $\frac{1}{2}$

(4) 2

71. Let $a, b \in \mathbb{R}, (a \neq 0)$. If the function f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & , 0 \leq x \leq 1 \\ a & , 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & , \sqrt{2} \leq x < \infty \end{cases}$$

Is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is:

(1) $(\sqrt{2}, 1 - \sqrt{3})$

(2) $(-\sqrt{2}, 1 + \sqrt{3})$

(3) $(\sqrt{2}, -1 + \sqrt{3})$

(4) $(-\sqrt{2}, 1 - \sqrt{3})$

72. Let $f(x) = \sin^4 x + \cos^4 x$. Then f is an increasing function in the interval :

(1) $\left] 0, \frac{\pi}{4} \right[$

(2) $\left[\frac{\pi}{4}, \frac{\pi}{2} \right[$

(3) $\left[\frac{\pi}{2}, \frac{5\pi}{8} \right[$

(4) $\left[\frac{5\pi}{8}, \frac{3\pi}{4} \right[$

73. Let C be a curve given by $y(x) = 1 + \sqrt{4x - 3}$, $x > \frac{3}{4}$. If P is a point on C , such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is:

(1) (2,3)

(2) (4,-3)

(3) (1,7)

(4) (3,-4)

74. The integral $\int \frac{dx}{(1 + \sqrt{x})\sqrt{x - x^2}}$ is equal to:

(where C is a constant of integration.)

(1) $-2\sqrt{\frac{1 + \sqrt{x}}{1 - \sqrt{x}}} + C$

$$(2) -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$$

$$(3) -\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$$

$$(4) 2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$$

75. The value of the integral $\int_4^{10} \frac{[x^2] dx}{[x^2 - 28x + 196] + [x^2]}$, where $[x]$ denotes the greatest integer less than or equal to x , is :

(1) 6

(2) 3

(3) 7

(4) $\frac{1}{3}$

76. For $x \in \mathbb{R}$, $x \neq 0$, if $y(x)$ is a differentiable function such that

$$x \int_1^x y(t) dt = (x+1) \int_1^x ty(t) dt, \text{ then } y(x) \text{ equals:}$$

(where C is a constant.)

(1) $\frac{C}{x} e^{-\frac{1}{x}}$

(2) $\frac{C}{x^2} e^{-\frac{1}{x}}$

(3) $\frac{C}{x^3} e^{-\frac{1}{x}}$

(4) $Cx^3 e^{-\frac{1}{x}}$

77. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{2} \sec x = \frac{\tan x}{2y}$,

where $0 \leq x \leq \frac{\pi}{2}$, and $y(0) = 1$, is given by:

(1) $y = 1 - \frac{x}{\sec x + \tan x}$

(2) $y^2 = 1 + \frac{x}{\sec x + \tan x}$

(3) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

(4) $y = 1 + \frac{x}{\sec x + \tan x}$

78. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point $(0, 1)$. The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is :

(1) $41x - 38y + 38 = 0$

(2) $41x - 25y + 25 = 0$

(3) $41x + 38y - 38 = 0$

(4) $41x - 25y + 25 = 0$

79. A straight line through origin O meets the lines $3y = 10 - 4x$ and $8x + 6y + 5 = 0$ at points A and B respectively. Then O divides the segment AB in the ratio :

(1) 2:3

(2) 1:2

(3) 4:1

(4) 3:4

80. Equation of the tangent to the circle, at the point $(1, -1)$, whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is :

(1) $4x + y - 3 = 0$

(2) $x + 4y + 3 = 0$

(3) $3x - y - 4 = 0$

(4) $x - 3y - 4 = 0$

81. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is :

(1) 2

(2) 4

(3) 6

(4) 8

82. A hyperbola whose transverse axis is along the major axis of the conic, $2 \frac{x^2}{3} + \frac{y^2}{4} = 4$ and has vertices at the foci of this conic. If

the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the

following points does NOT lie on it ?

(1) (0,2)

(2) $(\sqrt{5}, 2\sqrt{2})$

(3) $(\sqrt{10}, 2\sqrt{3})$

(4) $(5, 2\sqrt{3})$

83. ABC is a triangle in a plane with vertices A(2, 3, 5), B(-1, 3, 2) and C(λ , 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of ($\lambda^3 + \mu^3 + 5$) is :

(1) 1130

(2) 1348

(3) 676

(4) 1077

84. The number of distinct real values of λ for which the lines

$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar is:

(1) 4

(2) 1

(3) 2

(4) 3

85. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a} + \vec{b} + \vec{c}}{4}$ respectively,

then the position vector of the orthocentre of this triangle, is :

(1) $\vec{a} + \vec{b} + \vec{c}$

(2) $-\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$

(3) $\vec{0}$

(4) $\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$

86. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6 ; then the mean deviation from the mean of the data is :

(1) 2.4

(2) 2.8

(3) 2.5

(4) 2.6

87. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is :

(1) $\frac{240}{729}$

(2) $\frac{192}{729}$

(3) $\frac{256}{729}$

(4) $\frac{496}{729}$

88. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of

$\tan A + \tan B$ is :

(1) $\sqrt{3} - \sqrt{2}$

(2) $2 - \sqrt{3}$

(3) $4 - 2\sqrt{3}$

(4) $\frac{2}{\sqrt{3}}$

89. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45° . The angle of elevation of the top of the same tower from a point B, due south of A is 30° . If the distance between A and B is $54\sqrt{2}$ m , then the height of the tower (in metres), is :

- (1) $36\sqrt{3}$
- (2) 54
- (3) $54\sqrt{3}$
- (4) 108

90. The contrapositive of the following statement, “If the side of a square doubles, then its area increases four times”, is:

- (1) If the side of a square is not doubled, then its area does not increase four times.
- (2) If the area of a square increases four times, then its side is doubled.
- (3) If the area of a square increases four times, then its side is not doubled.
- (4) If the area of a square does not increase four times, then its side is not doubled.

PART-2

PREVIOUS YEAR PAPERS

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OTHER STATE EXAMS AND UNIVERSITY EXAM QUESTION PAPER

61. Solve for the set P as follows,

$$\rho(r) \propto \frac{1}{r}$$

Solve for the set Q as follows,

$$\frac{32}{23} \mu\text{F}$$

Therefore, both the sets are equal to each other.

62. The given equation is as follows,

$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

Rewrite the above equation and square both the sides.

400 Ω

Solve for $P_2 > P_1 > P_3$ at 7 Ω and 45° .

$$\frac{2E_0}{c} \hat{j} \cos kz \cos \omega t$$

63. The value of 27.5 cm is calculated as,

9 mm

It is given that the variable 10^{20} is a real number. Thus, the

imaginary part of $\frac{h^2}{4\pi m^2 r^3}$ will be zero.

4×10^{-2} gm

The complex number is written as follows,

6.9 mA

The sum of given geometric series is,

$$\lambda, P_{\text{eff}} = K \left(\frac{1}{\lambda} \right)^2$$

$$\frac{20}{3} \Omega$$

Further, substitute the above expression,
0.1 cm

$$P = a^{1/2} b^2 c^3 d^{-4}$$

Therefore, the required value is,

$$\frac{\Delta P}{P} = \left[\left(\frac{1}{2} \frac{\Delta a}{a} \right) + \left(2 \frac{\Delta b}{b} \right) + \left(3 \frac{\Delta c}{c} \right) + \left(4 \frac{\Delta d}{d} \right) \right] \times 100 \%$$

64. The given function is as follows.

$$\begin{aligned} \frac{\Delta P}{P} &= \left[\left(\frac{1}{2} \times 2 \right) + (2 \times 1) + (3 \times 3) + (4 \times 5) \right] \% \\ &= [1 + 2 + 9 + 20] \% \\ &= 32 \% \end{aligned}$$

Solve the given equation.

$$32 \%$$

Therefore, statement-I is true.

Solve the equation in statement-II,

$$s = ut + \frac{1}{2}at^2$$

Further solve the above equation.

$$s = (0)t + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}}$$

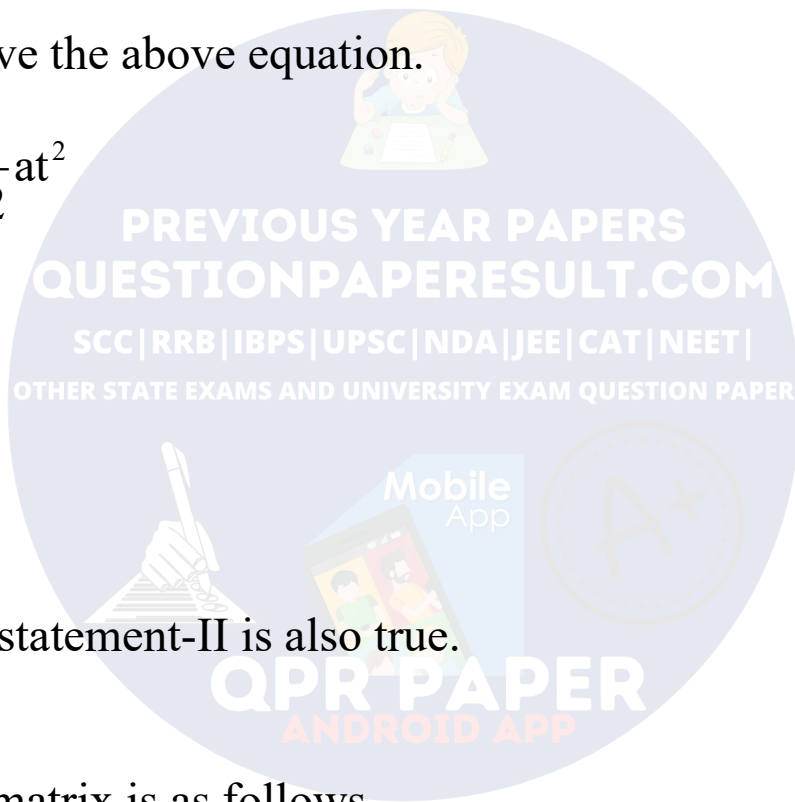
Therefore, statement-II is also true.

65. The given matrix is as follows,

$$(d + 200)$$

The determinant of given expression is calculated as,

$$t = \sqrt{\frac{2d}{2}} \quad \dots\dots (1)$$



The square of matrix A is calculated as,

$$(d + 200)$$

$$\sqrt{\frac{2d}{2}} = \sqrt{\frac{2(d + 200)}{4}}$$

Substitute $t = \sqrt{\frac{2(d + 200)}{4}}$ for $d = \frac{(d + 200)}{2}$ and

$$d = 200 \text{ m}$$

$$t = \sqrt{\frac{2(200 \text{ m})}{2}} \text{ for A in equation (1)}$$

$$= 10\sqrt{2} \text{ s}$$

$$10\sqrt{2} \text{ s}$$

66. The given expression is as follows,

$$M'$$

Solve the given expression.

$$v'$$

Further solve the above equation.

$$2M'v'\sin\theta = Mv\cos 45^\circ + Mv\cos 30^\circ$$

$$2M'v'\sin\theta = \frac{Mv}{\sqrt{2}} + \frac{\sqrt{3}Mv}{2}$$

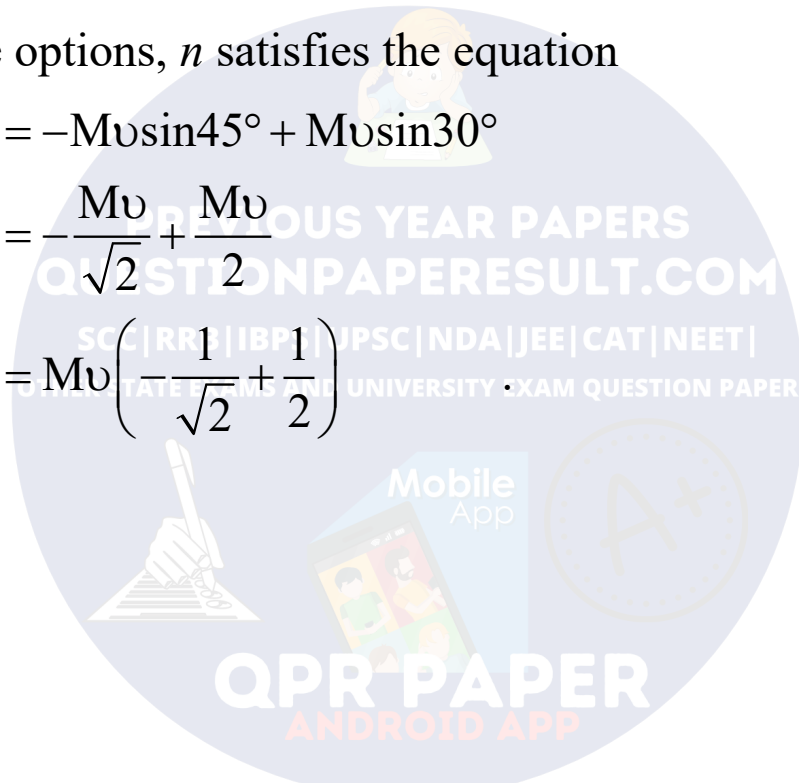
$$2M'v'\sin\theta = Mv\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)$$

Among the options, n satisfies the equation

$$2M'v'\cos\theta = -Mv\sin 45^\circ + Mv\sin 30^\circ$$

$$2M'v'\cos\theta = -\frac{Mv}{\sqrt{2}} + \frac{Mv}{2}$$

$$2M'v'\cos\theta = Mv\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$



$$\frac{2M'v'\sin\theta}{2M'v'\cos\theta} = \frac{Mv\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)}{Mv\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}$$

67. The expansion of the expression s

$$\tan\theta = \frac{\left(\frac{\sqrt{2} + \sqrt{3}}{2}\right)}{\left(\frac{1 - \sqrt{2}}{2}\right)}$$

$$\tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$$

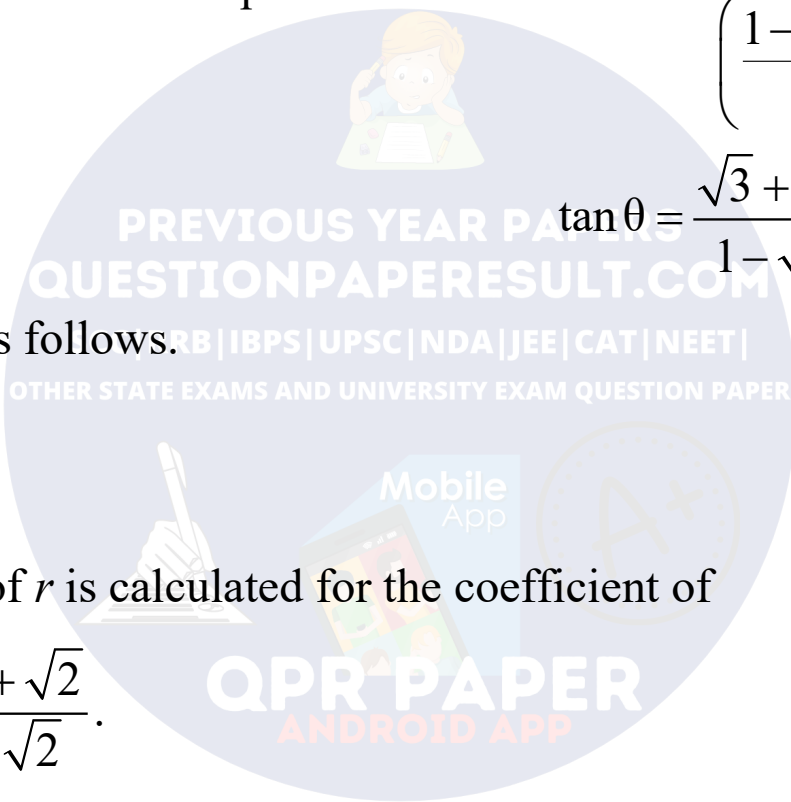
is written as follows.

θ

The value of r is calculated for the coefficient of

$$\tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}.$$

ΔPQR



The value of r is calculated for the coefficient of

$$h = \sqrt{1^2 - \left(\frac{x}{2}\right)^2}$$

$$= \frac{1}{2} \sqrt{4 - x^2}$$

$$v = \frac{dh}{dt}$$

The coefficient of $\frac{dh}{dt}$ and

$$\frac{dh}{dt} = \frac{d}{dt} \left(\frac{1}{2} \sqrt{4 - x^2} \right)$$

$$= \frac{1}{2} \frac{d}{dx} \left(\frac{1}{2} \sqrt{4 - x^2} \right) \frac{dx}{dt}$$

are m

$$= \frac{1}{4} \left(\frac{1}{\sqrt{4 - x^2}} \right) (-2x) \frac{dx}{dt}$$

$$= -\frac{x}{2\sqrt{4 - x^2}} \frac{dx}{dt}$$

and n respectively. So, the ratio is,

$$\frac{dh}{dt} = -\frac{1}{2\sqrt{\frac{4}{x^2} - 1}} \frac{dx}{dt}$$

68. The given series is as follows,

$$\sqrt{\frac{4}{x^2} - 1}$$

The numbers $\sqrt{\frac{4}{x^2} - 1}$ are the terms in an A.P. Thus,

$$\frac{dh}{dt}$$

The sum of first 17 terms of A.P. series is calculated as,

$$T \cos \theta = mg$$

69. The sum of expression is calculated as,

$$T \sin \theta = \frac{mv^2}{r}$$

Further solve the above equation.

$$\tan \theta = \frac{v^2}{rg}$$

70. The given expression with the limits is calculated as follows.

$$\tan 45^\circ = \frac{v^2}{(0.4 \text{ m})(10 \text{ ms}^{-2})}$$

$$v^2 = 4 \text{ m}^2 \text{s}^{-2}$$

$$v = \sqrt{4 \text{ m}^2 \text{s}^{-2}}$$

$$v = 2 \text{ ms}^{-1}$$

71. For the interval 2 ms^{-1} , the function $I_{\text{disc}} = \frac{MR^2}{2}$ is continuous.

Thus, the function is also continuous at

$$I_{\text{removed}} = \frac{1}{2} \left(\frac{M}{16} \right) \left(\frac{R^2}{16} \right) + \left(\frac{M}{16} \right) \left(\frac{9R^2}{16} \right)$$

$$= \frac{MR^2 + 18MR^2}{512}$$

$$= \frac{19MR^2}{512}$$

$$I_{\text{remaining}} = \frac{MR^2}{2} - \frac{19MR^2}{512}$$

$$= \frac{237MR^2}{512}$$

The limit of the function at $\frac{237MR^2}{512}$ is calculated as,

$$\rho = \frac{m}{v} = \frac{k}{r}$$

The limit of the function at $m = \frac{kv}{r}$ is calculated as,

$$\begin{aligned}g_{\text{inside}} &= \frac{Gmr}{R^3} \\ &= \left(\frac{Gr}{R^3}\right)\left(\frac{kv}{r}\right) \\ &= \frac{Gkv}{R^3}\end{aligned}$$

For $\frac{Gkv}{R^3}$, the value of b from above equation is calculated as,

$$g_{\text{out}} = \frac{Gm}{r^2}$$

For $F = Y\alpha_L A\Delta t$, the value of b is calculated as,

$$\begin{aligned}F &= (2 \times 10^{11} \text{ Nm}^{-2})(1.2 \times 10^{-5} \text{ K}^{-1})(40 \times 10^{-4} \text{ m}^2)(10) \\ &= 9.6 \times 10^4 \text{ N} \\ &= 1 \times 10^5 \text{ N}\end{aligned}$$

The value of b comes out to be an imaginary number which is not possible.

Therefore, the ordered pair $Q = \frac{\pi r^4 \Delta P}{8\eta L}$ is

$$\frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{\frac{l_1}{4}}$$

$$r_2^4 = \frac{r_1^4}{16}$$

$$r_2 = \frac{r_1}{2}$$

72. The given function is written as follows,

$$u_{\text{initial}} = \frac{5}{2} NRT$$

Differentiate the function $u_{\text{final}} = \frac{3}{2}(2nRT) + \frac{5}{2}(N-n)RT$ with respect to x .

$$= \frac{1}{2} nRT + \frac{5}{2} NRT$$

$$U_{\text{total}} = \frac{1}{2}nRT + \frac{5}{2}NRT - \frac{5}{2}NRT$$

$$= \frac{1}{2}nRT$$

Determine the interval for which the above function is increasing,

$$a = a_0 e^{\frac{-bt}{m}}$$

73. Let point $E \propto a^2$ be a point that lies on the curve C.
 $a \propto E$

Differentiate the given equation with respect to x .

$$a = \frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

$$\frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

$$= \frac{10^{-2}t}{0.1}$$

$$= \frac{t}{10}$$

$$\frac{a_0}{\sqrt{2}} = a_0 e^{-\frac{t}{10}}$$

Thus, the point P is obtained as $\frac{1}{\sqrt{2}} = e^{-\frac{t}{10}}$.

$$\ln \sqrt{2} = \frac{t}{10}$$

$$t = 3.5 \text{ sec}$$

The slope of normal at point P $v = \frac{\omega}{k}$ is equal to $v = \frac{200\pi}{\left(\frac{5\pi}{4}\right)} = 160 \text{ m/s}$.

The equation of normal is calculated as follows,

$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

Among all the given options, it is clear that normal passes

through the point $C = \frac{\epsilon_0 A}{3}$.

74. Integrate the given expression.

$$C = \frac{\left(\frac{k\epsilon_0 A}{3}\right)\left(\frac{\epsilon_0 A}{2.4}\right)}{\frac{k\epsilon_0 A}{3} + \frac{\epsilon_0 A}{2.4}}$$

$$\frac{\epsilon_0 A}{3} = \frac{\left(\frac{k\epsilon_0 A}{3}\right)\left(\frac{\epsilon_0 A}{2.4}\right)}{\frac{k\epsilon_0 A}{3} + \frac{\epsilon_0 A}{2.4}}$$

Substitute $3k = 2.4k + 3$ for $\sigma_1 = \epsilon_0 \nu B$, $\sigma_2 = -\epsilon_0 \nu B$ and $k = 5$

then differentiate of this expression.

$$\frac{(A - C)}{D}$$

Substitute these values in the given expression and then integrate.

$$MnR^2t$$

Further solve the above expression.

$$-4500 J \quad \dots\dots (1)$$

The value of cosine in terms of x is calculated as,

1.37

Substitute this value in equation (1).

$$F_G = \frac{GMm}{(R + h)^2}$$

75. The given integral is as follows,

$$(4\pi\mu Bb)\Delta n \dots\dots (1)$$

Rewrite the equation (1) by using the following property,

$$1.67 \times 10^5 \text{ J}$$

Thus, equation (1) becomes,

$$1.9 \text{ Hz} \dots\dots (2)$$

Add equation (1) and equation (2).

$$170 \text{ Hz}$$

76. The given integral is written as follows,

$$\rho(r) \propto \frac{1}{r}$$

Differentiate above equation with respect to x .

$$\frac{32}{23} \mu\text{F}$$

Differentiate the above equation.

$$\sigma_1 = \epsilon_0 v B, \sigma_2 = -\epsilon_0 v B$$

Further solve the above differential equation.

$$400 \Omega$$

77. The given differential equation is written as follows,

$$P_2 > P_1 > P_3 \dots\dots (1)$$

Substitute 7Ω and 45° for $\frac{2E_0}{c} \hat{j} \cos kz \cos \omega t$.

Differentiate 27.5 cm with respect to x ,

$$9 \text{ mm}$$

Substitute the values in equation (1).

$$10^{20} \dots (2)$$

The integrated factor of the above equation is,

$$\frac{h^2}{4\pi m^2 r^3}$$

The solution of differential equation (2) is calculated as follows,

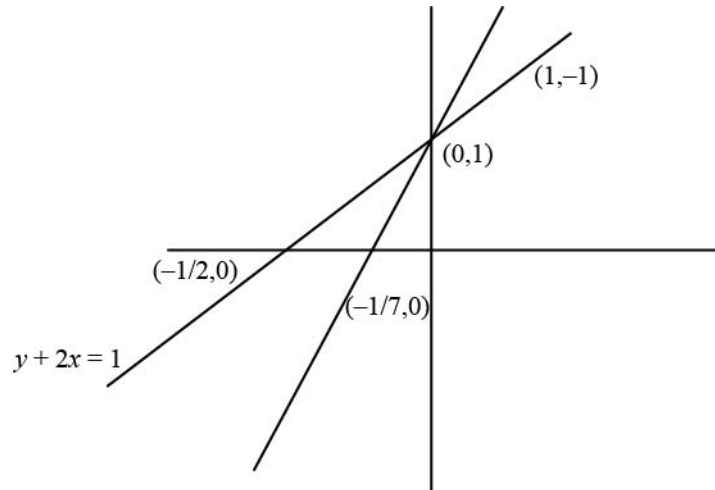
$$4 \times 10^{-2} \text{ gm}$$

78. The equation of incident line is as follows,

$$6.9 \text{ mA} \dots (1)$$

Let a point $\lambda, P_{\text{eff}} = K \left(\frac{1}{\lambda} \right)^2$ be on the line $\frac{20}{3} \Omega$ and the image of 0.1 cm lie on the incidence line in $P = a^{1/2} b^2 c^3 d^{-4}$.

The following figure shows the incident line.



The equation of line is given by,

$$\frac{\Delta P}{P} = \left[\left(\frac{1}{2} \frac{\Delta a}{a} \right) + \left(2 \frac{\Delta b}{b} \right) + \left(3 \frac{\Delta c}{c} \right) + \left(4 \frac{\Delta d}{d} \right) \right] \times 100\%$$

From the above equation, the value of x is calculated as,

$$\begin{aligned} \frac{\Delta P}{P} &= \left[\left(\frac{1}{2} \times 2 \right) + (2 \times 1) + (3 \times 3) + (4 \times 5) \right] \% \\ &= [1 + 2 + 9 + 20] \% \\ &= 32 \% \end{aligned}$$

The value of y is calculated as,

$$32 \%$$

Substitute the values of x and y in equation (1).

$$s = ut + \frac{1}{2}at^2$$

$$s = (0)t + \frac{1}{2}at^2$$

Substitute $s = \frac{1}{2}at^2$ for $(d + 200)$ in equation (1).

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{2d}{2}}$$

79. Let the straight line through origin $(d + 200)$ be given by,

$$t = \sqrt{\frac{2(d + 200)}{4}}$$

$$\sqrt{\frac{2d}{2}} = \sqrt{\frac{2(d + 200)}{4}}$$

The above line intersects the line $d = \frac{(d + 200)}{2}$ at point

$$d = 200 \text{ m}$$

A, then,

$$t = \sqrt{\frac{2(200 \text{ m})}{2}} \quad \dots\dots (1)$$

$$= 10\sqrt{2} \text{ s}$$

Again the line through the origin meets line $10\sqrt{2} \text{ s}$ at point B

thus,

$$M' \quad \dots\dots (2)$$

Divide equation (1) by equation (2),

$$v'$$

80. The equation of the first straight line is,

$$2M'v'\sin\theta = Mv\cos 45^\circ + Mv\cos 30^\circ$$

$$2M'v'\sin\theta = \frac{Mv}{\sqrt{2}} + \frac{\sqrt{3}Mv}{2}$$

$$2M'v'\sin\theta = Mv \left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \right)$$

$$\dots\dots (1)$$

The equation of the second straight line is,

$$2M'v'\cos\theta = -Mv\sin 45^\circ + Mv\sin 30^\circ$$

$$2M'v'\cos\theta = -\frac{Mv}{\sqrt{2}} + \frac{Mv}{2}$$

$$2M'v'\cos\theta = Mv\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

..... (2)

Add both equation (1) and (2) to obtain the value of x as follows.

$$\frac{2M'v'\sin\theta}{2M'v'\cos\theta} = \frac{Mv\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)}{Mv\left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}$$

$$\tan\theta = \frac{\left(\frac{\sqrt{2} + \sqrt{3}}{2}\right)}{\left(\frac{1 - \sqrt{2}}{2}\right)}$$

$$\tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$$

Substitute θ for x in equation (1).

$$\tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$$

The centre of circle is the point where the straight lines ΔPQR

and $h = \sqrt{1^2 - \left(\frac{x}{2}\right)^2}$ intersect. Thus, the centre of circle is at

$$= \frac{1}{2} \sqrt{4 - x^2}$$

$$v = \frac{dh}{dt}.$$

The equation of the circle is calculated as,

$$\frac{dh}{dt}$$

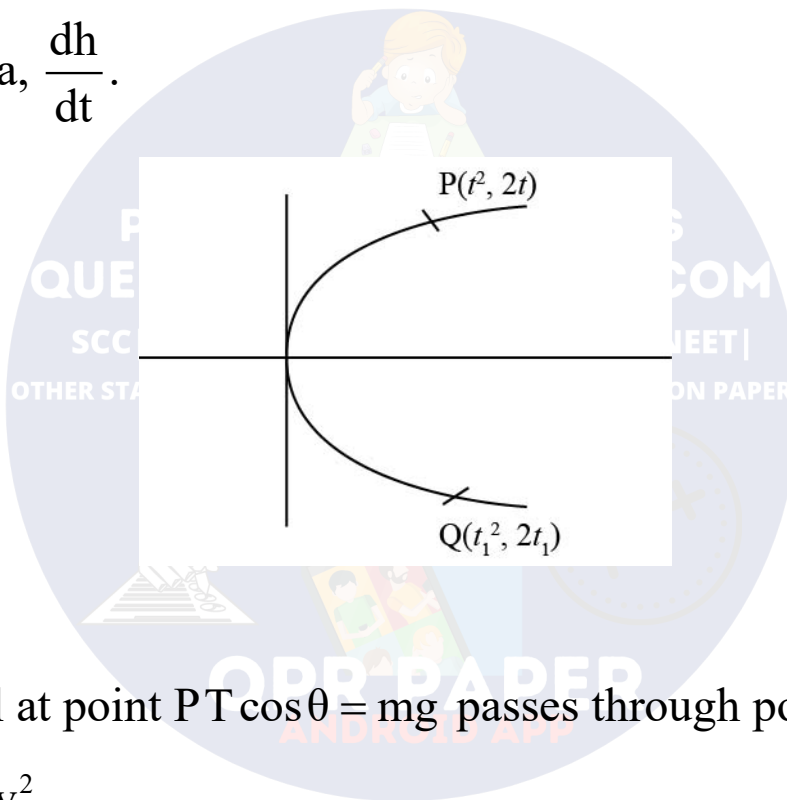
The equation of tangent to the circle at point

$$\begin{aligned} \frac{dh}{dt} &= \frac{d}{dt} \left(\frac{1}{2} \sqrt{4 - x^2} \right) \\ &= \frac{1}{2} \frac{d}{dx} \left(\frac{1}{2} \sqrt{4 - x^2} \right) \frac{dx}{dt} \\ &= \frac{1}{4} \left(\frac{1}{\sqrt{4 - x^2}} \right) (-2x) \frac{dx}{dt} \\ &= - \frac{x}{2\sqrt{4 - x^2}} \frac{dx}{dt} \end{aligned}$$

is calculated as,

$$\frac{dh}{dt} = -\frac{1}{2\sqrt{\frac{4}{x^2}-1}} \frac{dx}{dt}$$

81. Consider point P as $\sqrt{\frac{4}{x^2}-1}$ and consider point Q as $\sqrt{\frac{4}{x^2}-1}$ on the parabola, $\frac{dh}{dt}$.



The normal at point P $T \cos \theta = mg$ passes through point Q

$T \sin \theta = \frac{mv^2}{r}$ and the equation is given as,

$$\tan \theta = \frac{v^2}{rg} \quad \dots\dots (1)$$

Differentiate the above equation with respect to t .

$$\tan 45^\circ = \frac{v^2}{(0.4 \text{ m})(10 \text{ ms}^{-2})}$$

$$v^2 = 4 \text{ m}^2 \text{s}^{-2}$$

$$v = \sqrt{4 \text{ m}^2 \text{s}^{-2}}$$

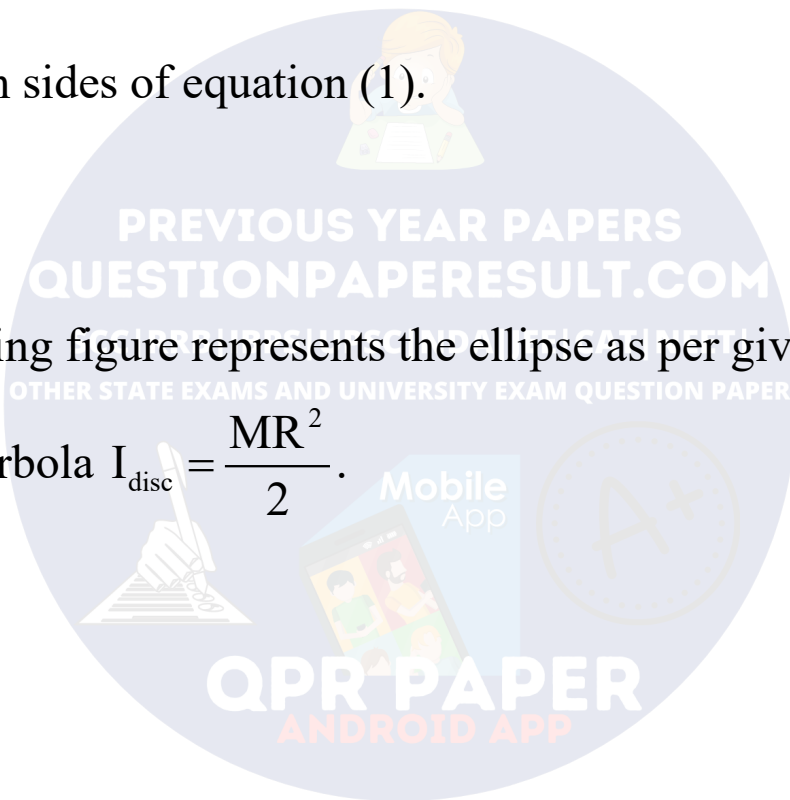
$$v = 2 \text{ ms}^{-1}$$

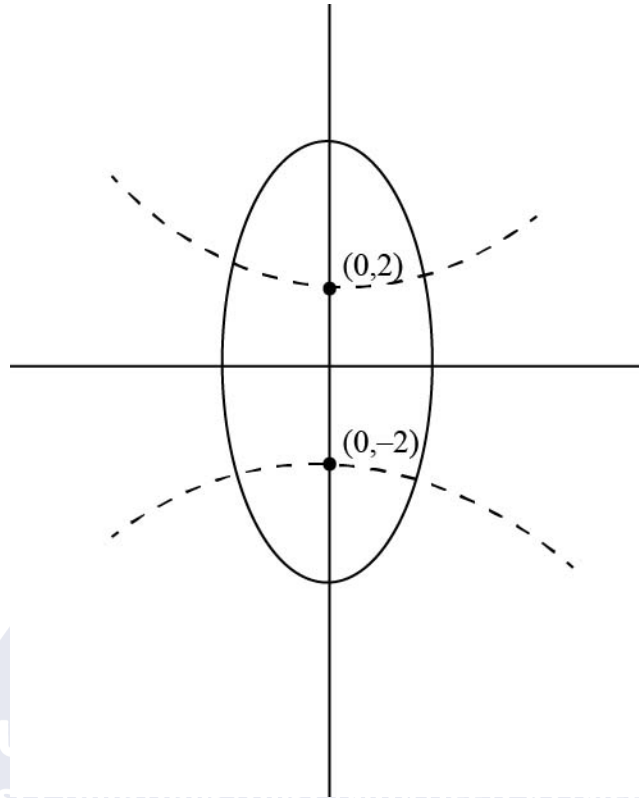
Square both sides of equation (1):

$$2 \text{ ms}^{-1}$$

82. The following figure represents the ellipse as per given the point

of the hyperbola $I_{\text{disc}} = \frac{MR^2}{2}$.





For ellipse,

The equation of ellipse is written as,

$$\begin{aligned}
 I_{\text{removed}} &= \frac{1}{2} \left(\frac{M}{16} \right) \left(\frac{R^2}{16} \right) + \left(\frac{M}{16} \right) \left(\frac{9R^2}{16} \right) \\
 &= \frac{MR^2 + 18MR^2}{512} \\
 &= \frac{19MR^2}{512}
 \end{aligned}$$

Let the foci of ellipse be $I_{\text{remaining}} = \frac{MR^2}{2} - \frac{19MR^2}{512}$. Thus, the

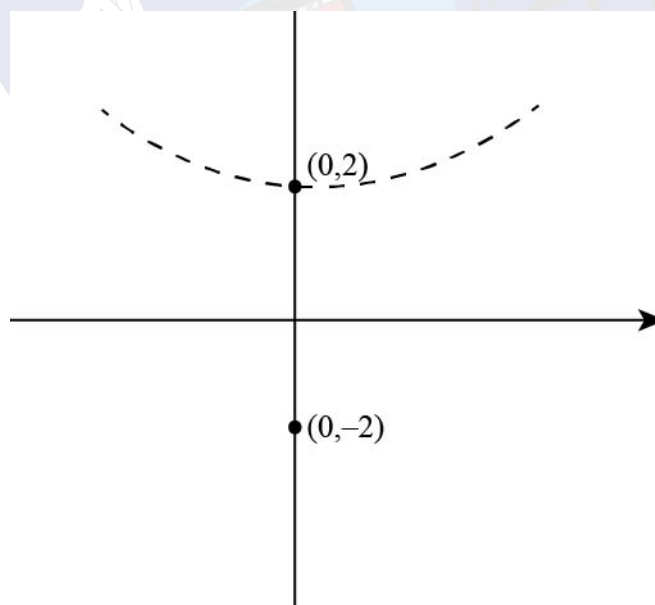
$$= \frac{237MR^2}{512}$$

eccentricity of ellipse is calculated as,

$$\frac{237MR^2}{512}$$

Thus, the point $\rho = \frac{m}{v} = \frac{k}{r}$ or $m = \frac{kv}{r}$ does not lie on the parabola.

For hyperbola,



Equation of the hyperbola is given by,

$$\begin{aligned}
 g_{\text{inside}} &= \frac{Gmr}{R^3} \\
 &= \left(\frac{Gr}{R^3} \right) \left(\frac{kv}{r} \right) \\
 &= \frac{Gkv}{R^3}
 \end{aligned}$$

The eccentricity of the hyperbola is calculated as,

$$\frac{Gkv}{R^3}$$

Given that $g_{\text{out}} = \frac{Gm}{r^2}$, therefore,

$$F = Y\alpha_L A\Delta t$$

Thus, the equation of the ellipse is,

$$\begin{aligned}
 F &= (2 \times 10^{11} \text{ Nm}^{-2})(1.2 \times 10^{-5} \text{ K}^{-1})(40 \times 10^{-4} \text{ m}^2)(10) \\
 &= 9.6 \times 10^4 \text{ N} \\
 &= 1 \times 10^5 \text{ N}
 \end{aligned}$$

All the options are checked by substituting the given points on the hyperbola,

Check option (4).

$$\text{For the point } Q = \frac{\pi r^4}{8\eta} \frac{\Delta P}{L},$$

$$\frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

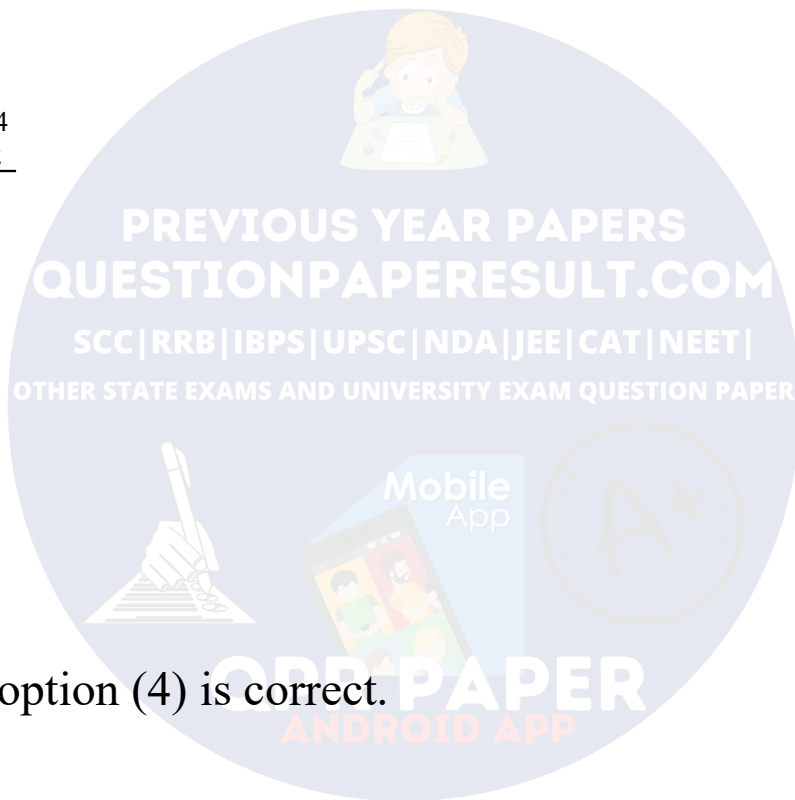
$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{\frac{l_1}{4}}$$

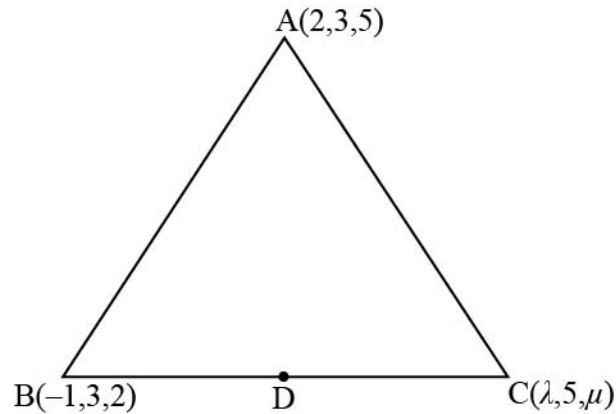
$$r_2^4 = \frac{r_1^4}{16}$$

$$r_2 = \frac{r_1}{2}$$

Therefore, option (4) is correct.

83. The following figure shows the triangle ABC with all its vertices.





The coordinates of point D is as follows,

$$u_{\text{initial}} = \frac{5}{2}NRT$$

The direction cosine of median AD is given by,

$$\begin{aligned} u_{\text{final}} &= \frac{3}{2}(2nRT) + \frac{5}{2}(N-n)RT \\ &= \frac{1}{2}nRT + \frac{5}{2}NRT \end{aligned}$$

The vector AD is written as follows,

$$\begin{aligned} U_{\text{total}} &= \frac{1}{2}nRT + \frac{5}{2}NRT - \frac{5}{2}NRT \\ &= \frac{1}{2}nRT \end{aligned}$$

From the above expression,

$$a = a_0 e^{\frac{-bt}{m}}$$

From the above expression, the value of $E \propto a^2$ is found out to
 $a \propto E$

$$a = \frac{a_0}{\sqrt{2}} = \frac{bt}{m}$$

be 7 and the value of $\frac{a_0}{\sqrt{2}} = \frac{bt}{m}$ is found out to be 10.

$$= \frac{10^{-2} t}{0.1}$$

$$= \frac{t}{10}$$

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Thus, the value of given expression is calculated as,

$$\frac{a_0}{\sqrt{2}} = a_0 e^{-\frac{t}{10}}$$

$$\frac{1}{\sqrt{2}} = e^{-\frac{t}{10}}$$

$$\ln \sqrt{2} = \frac{t}{10}$$

$$t = 3.5 \text{ sec}$$

84. The given equation of line is as follows,

$$v = \frac{\omega}{k}$$

$$v = \frac{200\pi}{\left(\frac{5\pi}{4}\right)}$$

$$= 160 \text{ m/s}$$

The given two lines are coplanar. Thus,

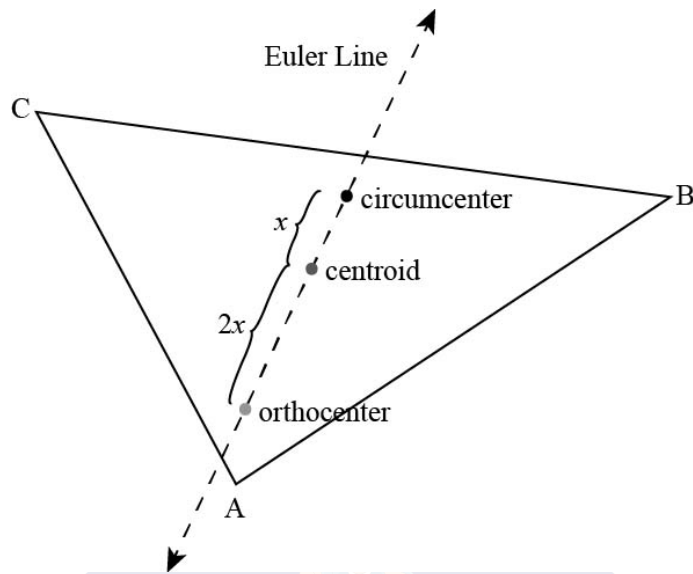
$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

Therefore, there are three possible values of λ .

85. The expression for the centroid of the triangle ABC for the given position vectors \vec{a} , \vec{b} and \vec{c} is as follows,

$$\text{Centroid} \equiv \left(\frac{\vec{a} + \vec{b} + \vec{c}}{3} \right)$$

The relationship between the centroid, circumcenter and orthocenter is shown in the following diagram.



Therefore,

$$\text{Orthocentre} = 3(\text{centroid}) - 2(\text{circumcenter})$$

$$\text{Orthocentre} = 3\left(\frac{\vec{a} + \vec{b} + \vec{c}}{3}\right) - 2\left(\frac{\vec{a} + \vec{b} + \vec{c}}{3}\right)$$

$$= \left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$$

86. The mean of 5 observations is 5. This can be expressed in the numerical form as,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$5 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 25$$

$$\sum_{i=1}^5 x_i = 25 \quad \dots\dots (1)$$

The variance of the observations is calculated as,

$$\sigma^2 = 124$$

$$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 124$$

$$\sum x_i^2 = 745$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 745$$

Substitute the given three observations,

Consider as the three observations x_3 , x_4 and x_5 are 1, 2 and 6 respectively.

Therefore, from the above equation,

$$x_1^2 + x_2^2 + (1^2) + 2^2 + 6^2 = 745$$

$$x_1^2 + x_2^2 = 704 \quad \dots\dots (2)$$

Similarly,

From equation (1),

$$x_1 + x_2 + 1 + 2 + 6 = 25$$

$$x_1 + x_2 = 16 \quad \dots\dots (3)$$

From equation (2) and equation (3),

$$(x_1 + x_2)^2 = (16)^2$$

$$2x_1x_2 + 704 = 256$$

$$x_1x_2 = \frac{256 - 704}{2}$$

$$x_1x_2 = -224$$

The mean deviation is calculated as follows,

$$\frac{\sum |x_i - 5|}{5} = \frac{|x_1 - 5| + |x_2 - 5| + |1 - 5| + |2 - 5| + |6 - 5|}{5}$$

$$= \frac{8 + |x_1 - 5| + |16 - x_1 - 5|}{5}$$

$$= \frac{8 + 6}{5}$$

$$= 2.8$$

87. The experiment succeeds twice as often as it fails. The condition is mathematically written as,

$$p = 2q$$

The total number of experiments is given by,

$$p + q = 1$$

From above equations, the value of q is $\frac{1}{3}$ and the value of p is

$$\frac{2}{3}.$$

The probability of at least 5 successes in the six trials is given by,

$$\begin{aligned} {}^6C_5 p^5 q + {}^6C_6 p^6 &= 6 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + 1 \left(\frac{2}{3}\right)^6 \\ &= \frac{256}{729} \end{aligned}$$

88. Consider the given expression as,

$$y = \tan A + \tan B$$

Differentiate the above equation with respect to A .

$$\begin{aligned}\frac{dy}{dA} &= \sec^2 A - \sec^2 B \\ &= \sec^2 A - \sec^2 \left(\frac{\pi}{6} - A \right) \quad \left(\because A + B = \frac{\pi}{6} \right)\end{aligned}$$

The expression $(\tan A + \tan B)$ increases in the interval $\left[\frac{\pi}{12}, \frac{\pi}{6} \right]$

and decreases in the interval $\left[0, \frac{\pi}{12} \right]$. Thus, it can be concluded

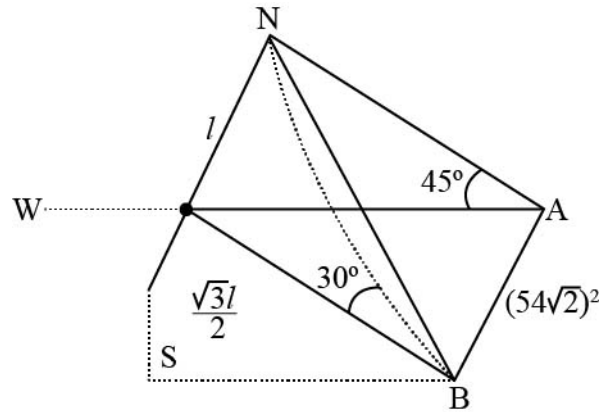
that the expression $(\tan A + \tan B)$ is minimum when A and B is

equal to $\frac{\pi}{12}$.

Therefore, the minimum value of the expression is calculated as,

$$\begin{aligned}y_{\min} &= 2 \tan \frac{\pi}{12} \\ &= (2 - \sqrt{3})2 \\ &= 4 - 2\sqrt{3}\end{aligned}$$

89. The following figure is a pictorial representation of the given conditions.



From the figure the height of the tower is calculated as,

$$\frac{l^2}{4} + (54\sqrt{2})^2 = \frac{3l^2}{4}$$

$$(54)^2 \times 2 \times 2 = l^2$$

$$l = 54 \times 2$$

$$= 108 \text{ m}$$

90. Assume the side of a square doubles to be represented as p . The area of square increases by four times. Let it be represents by q .

Thus, the contra positive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$.

Therefore, the correct option is (4).