IIT JEE MAINS 2016 9TH APRIL 2016

MATHEMATICS

61. For $x \in \mathbf{R}, x \neq 0, x \neq 1,$ let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x)), n = 0, 1, 2, \ldots$ Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to :

(1) $\frac{8}{3}$

(2) $\frac{5}{3}$

(3) $\frac{4}{3}$

(4) $\frac{1}{3}$

62. The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :

(1) $2+2i$

(2) $1+i$

(3) $-1-i$

(4) $-2-2i$

63. If the equations $x^2+bx-1=0$ and $x^2+x+b=0$ have a common root different from -1, then $|b|$ is equal to :

(1) $\sqrt{2}$

(2) 2

(3) 3

(4) $\sqrt{3}$

64. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$ then $P^TQ^{2015}P$ is :

(1) $\begin{bmatrix}0 & 2015\\ 0 & 0\end{bmatrix}$

(2) $\begin{bmatrix}2015 & 1\\ 0 & 2015\end{bmatrix}$

(3) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

65. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is :

(1) 4

(2) 3

(3) 2

(4) 1

66. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is :

(1) $\frac{11!}{(2!)^3}$

(2) 110

(3) 56

(4) 59

67. For $x \in \mathbf{R}, x \neq -1$, if

$$(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + ... + x^{2016} = \sum_{i=0}^{2016} a_i x^i$$, then a_{17} is equal to :

(1) $\frac{2017!}{17!2000!}$

(2) $\frac{2016!}{17!1999!}$

(3) $\frac{2017!}{2000!}$

(4) $\frac{2016!}{16!}$

68. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)\cdot(600)^3$. Then $x^3 + y^3 + z^3$ is equal to :

(1) 270

(2) 258

(3) 342

(4) 216

69. The value of $\sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}} \right)$ is equal to :

(1) 560

(2) 680

(3) 1240

(4) 1085

70. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2} \right)^{2x} = e^3$, then $'a'$ is equal to :

(1) 2

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{1}{2}$

71. If the function $f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x+b), & 1 \le x \le 2 \end{cases}$ is differentiable at $x = 1$, then $\frac{a}{b}$ is equal to :

(1) $\dfrac{\pi-2}{2}$

(2) $\dfrac{-\pi-2}{2}$

(3) $\dfrac{\pi+2}{2}$

(4) $-1-\cos^{-1}(2)$

72. If the tangent at a point P, with parameter t, on the curve $x=4t^2+3$, $y=8t^3-1$, $t \in \mathbf{R}$, meets the curve again at a point Q, then the coordinates of Q are :

(1) $(t^2+3, -t^3-1)$

(2) $(4t^2+3, -8t^3-1)$

(3) (t^2+3, t^3-1)

(4) $(16t^2+3, -64t^3-1)$

73. The minimum distance of a point on the curve $y=x^2-4$ from the origin is :

(1) $\dfrac{\sqrt{19}}{2}$

(2) $\sqrt{\dfrac{15}{2}}$

(3) $\frac{\sqrt{15}}{2}$

(4) $\sqrt{\frac{19}{2}}$

74. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^{A} + C(\tan x)^{B} + k$, where k is a constant of integration, then A+B+C equals :

(1) $\frac{21}{5}$

(2) $\frac{16}{5}$

(3) $\frac{7}{10}$

(4) $\frac{27}{10}$

75. If $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1-x+x^2)dx$, then $\int_0^1 \tan^{-1}(1-x+x^2)dx$ is equal to :

(1) $\log 4$

(2) $\frac{\pi}{2} + \log 2$

(3) $\log 2$

(4) $\frac{\pi}{2} - \log 4$

76. The area (in sq. units) of the region described by $A = \left\{ (x, y) \middle| y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0 \right\}$ is :

(1) $\frac{7}{2}$

(2) $\frac{19}{6}$

(3) $\frac{13}{6}$

(4) $\frac{17}{6}$

77. If $f(x)$ is a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$, then $f\left(3/2\right)$ is equal to :

(1) $\frac{13}{6}$

(2) $\frac{23}{18}$

(3) $\frac{25}{9}$

(4) $\frac{31}{18}$

78. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is :

(1) $6xy=7(x+y)$

(2) $4(x+y)2-28(x+y)+49=0$

(3) $7xy=6(x+y)$

(4) $14(x+y)^2-97(x+y)+168=0$

79. The point (2, 1) is translated parallel to the line L : $x-y=4$ by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is:

(1) $x+y=2-\sqrt{6}$

(2) $x+y=3-3\sqrt{6}$

(3) $x+y=3-2\sqrt{6}$

(4) $2x+2y=1-\sqrt{6}$

80. A circle passes through (−2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle ?

(1) $4x+5y-6=0$

(2) $2x-3y+10=0$

(3) $3x+4y-3=0$

(4) $5x+2y+4=0$

81. Let a and b respectively be the semitransverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2-18e+5=0$. If S(5, 0) is a focus and $5x=9$ is the corresponding directrix of this hyperbola, then a^2-b^2 is equal to:

(1) 7

(2) −7

(3) 5

(4) −5

82. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is :

(1) $\frac{9}{2}$

(2) $3\sqrt{3}$

(3) $9\sqrt{3}$

(4) 9

83. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval :

(1) [0, 1)

(2) [1, 2)

(3) (2, 3]

(4) (3, 4]

84. The distance of the point (1, −2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x-y+2z=3$ and $2x-2y+z+12=0$, is :

(1) $2\sqrt{2}$

(2) 2

(3) $\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

85. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}, -\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point ($p$, q) lies on a line :

(1) parallel to x-axis.

(2) parallel to y-axis.

(3) making an acute angle with the positive direction of x-axis.

(4) making an obtuse angle with the positive direction of x-axis.

86. If the mean deviation of the numbers 1, 1+d, ..., 1+100d from their mean is 255, then a value of d is :

(1) 10.1

(2) 20.2

(3) 10

(4) 5.05

87. If A and B are any two events such that $P(A)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$, then the conditional probability, $P(A|(A'\cup B'))$, where A' denotes the complement of A, is equal to :

(1) $\frac{1}{4}$

(2) $\frac{5}{17}$

(3) $\frac{8}{17}$

(4) $\frac{11}{20}$

88. The number of $x\in[0,2\pi]$ for which $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$ is :

(1) 2

(2) 4

(3) 6

(4) 8

89. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x\in\mathbf{R}$, then $M-m$ is equal to :

(1) $\frac{15}{4}$

(2) $\frac{9}{4}$

(3) $\frac{7}{4}$

(4) $\frac{1}{4}$

90. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals :

(1) (T, T)

(2) (T, F)

(3) (F, T)

(4) (F, F)

PART-2

61.Consider the given fuction at $n=0$,

$$f_0(x)=\frac{1}{1-x}$$

And also at $(n+1)$ the expression of the fuction is,

$$f_{n+1}(x)=f_0\left(f_n(x)\right)$$

Now, substitute $n=0$ in above function $f_{n+1}(x)$,

$$f_{0+1}(x)=f_0\left(f_0(x)\right)$$
$$f_1(x)=f_0\left(f_0(x)\right)$$

Substitute the given value in the fuction,

$$f_1(x)=\frac{1}{1-f_0(x)};f_0(x)\neq 1$$
$$=\frac{1}{1-\frac{1}{1-x}};x\neq 0$$
$$=\frac{1-x}{-x}$$
$$=1-\frac{1}{x}$$

Again, substitute $n=1$ in above function $f_{n+1}(x)$,

$$f_{1+1}(x) = f_0(f_1(x))$$

$$f_2(x) = f_0(f_1(x))$$

Substitute the value in fuction $f_2(x)$,

$$f_2(x) = f_0(f_1(x))$$

$$= \frac{1}{1 - f_1(x)}; f_1(x) \neq 1$$

$$= \frac{1}{1 + \frac{1-x}{x}}$$

$$= x$$

Similarly substitute the value $n = 2, 3, 4, \ldots$ in the function $f_{n+1}(x)$,

$$f_3(x) = f_0(x)$$

$$f_4(x) = f_1(x)$$

.

$$f_{100}(x) = f_1(x)$$

Therefore, the required value is,

$$f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = f_1(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$$

Substitute the values in the above expression,

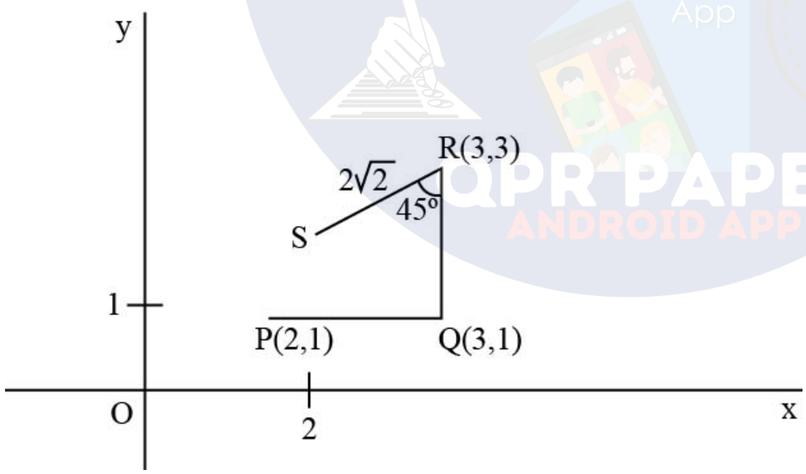
$$=\left(1-\frac{1}{3}\right)+\left(1-\frac{3}{2}\right)+\left(\frac{3}{2}\right)$$

$$=\frac{5}{3}$$

Hence, the required value is equal to $\frac{5}{3}$.

62. Let P represent the point $(2+i)$.

It is given that, the point $(2+i)$ moves 1 unit eastward. It means x coordinate will become 3. It again moves 2 unit northward, it means y coordinate will become $3i$. Hence, new location would be $3+3i$. The required diagram is shown below,



By using rotation theorem, find the coordinate,

$$\frac{z-(3+3i)}{3+i-(3+3i)}=\frac{2\sqrt{2}}{2}e^{-\pi i/4}$$

$$\frac{z-3-3i}{-2i}=1-i$$

$$z-3-3i=-2i-2$$

$$z=1+i$$

63. Consider the given equation:

$$x^2+bx-1=0$$

$$x^2+x+b=0$$

These equations have a common root. Hence, α will satisfy the given equation as,

$$\alpha^2+b\alpha-1=0$$

$$\alpha^2+\alpha+b=0$$

Hence, for the roots:

$$\frac{\alpha^2}{b^2+1}=\frac{\alpha}{-(b+1)}=\frac{1}{(1-b)}$$

$$(b+1)^2=(b^2+1)(1-b)$$

$$b^2+2b+1=b^2-b^3+1-b$$

$$b^3+3b=0$$

Further solve the above equation to calculate the value of b.

$b^3 + 3b = 0$

$b = 0$ or $b^2 = -3$

When $b = 0$, then common root is (-1). Hence, $b = 0$ does not satisfy the condition.

Hence,

$b^2 = -3$

$b = \pm\sqrt{3}i$

$|b| = \sqrt{3}$

64. Consider the given matrix,

$$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

According to the property of matrices:

$PP^T = P^TP$

The matrix PP^T is,

$$PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= P^T P$$

Now,

$$P^T Q^{2015} P = P^T PAP^T PAP^T \text{.........} PAP^T \text{.....2015 times}$$

Because A^{2015}.

Calculate the value of A^{2015}.

$$A^2 - 2A + 1 = 0$$

$$A^n = nA - (n-1)1$$

$$A^{2015} = 2015 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 2014 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$$

65. Given that the condition for the district real roots is,

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Simplify the matrix by row column method,

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} \cos x + 2\sin x & \cos x + 2\sin x & \cos x + 2\sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Take out $(\cos x + 2\sin x)$ common from the first row (R_1),

$$(\cos x + 2\sin x)\begin{vmatrix} 1 & 1 & 1 \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Apply $C_1 \to C_1 - C_3$ to make two zeros in the first column and then expand,

$$(\cos x + 2\sin x)\begin{vmatrix} 0 & 1 & 1 \\ 0 & \cos x & \sin x \\ \sin x - \cos x & \sin x & \cos x \end{vmatrix} = 0$$

$$(\cos x + 2\sin x)\left((\sin x - \cos x)(\cos x - \sin x)\right) = 0$$

$$(\cos x + 2\sin x)(\cos x - \sin x)^2 = 0$$

Further, simplify the above equation:

For $(\cos x + 2\sin x) = 0$,

$$(\cos x + 2\sin x) = 0$$

$$\cos x = -2\sin x$$

$$\tan x = \frac{-1}{2}$$

$$x = \tan^{-1}\left(\frac{-1}{2}\right)$$

And,

For $(\cos x - \sin x)^2 = 0$,

$$(\cos x - \sin x)^2 = 0$$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}$$

Hence, it has two solutions.

66. The given word 'MEDITERRANEAN' consists of 13 letters.

The numbers of the letters are,

$M = 1$

$E = 3$

$D = 1$

$I = 1$

And,

$T = 1$

$R = 2$

$A = 2$

$N = 2$

Here, the number of different letters is 8.

Given that the position of E and R is fixed

Therefore, rest of 11 letters can be arranged in,

$$\frac{11!}{(2!)\times(2!)\times(2!)} = \frac{11!}{(2!)^3}$$

67. Simplify the given equation:

$$\sum_{i=0}^{2016} C_i x^i = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + ... + x^{2016}$$

$$= \frac{(1+x)^{2016}\left(1-\left(\frac{x}{1+x}\right)^{2017}\right)}{1-\frac{x}{1+x}}$$

$$= \frac{\frac{(1+x)^{2016}}{1} - \frac{x^{2017}}{(1+x)}}{\frac{x+1-x}{1+x}}$$

$$= \frac{(1+x)^{2017} - x^{2017}}{1}$$

Further simplify the above equation:

$$a_{17} = {}^{2017}C_{17} \qquad \left[\because {}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}\right]$$

$$= \frac{2017!}{17!(2017-17)!}$$

$$= \frac{2017!}{17!2000!}$$

68. Consider the given function:

$$x + y + z = 12$$

$$x^3 y^4 z^5 = (0.1)(600)^3$$

The relation between AM, GM, and HM of two positive numbers is,

$$\text{AM} \geq \text{GM} \geq \text{HM}$$

Therefore,

$$\text{AM} \geq \text{GM}$$

Substitute the value,

$$\frac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12} \geq \left(\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^3\left(\frac{z}{5}\right)^3\right)^{1/12}$$

$$\frac{12}{12} \geq \frac{x^3y^4z^5}{(60)^3(4\times 25)}$$

$$(0.1)(600)^3 \geq x^3y^4z^5$$

$$x^3y^4z^5 \leq (0.1)(600)^3$$

But the given condition is,

$$x^3y^4z^5 = (0.1)(600)^3$$

Therefore, it is clear that,

$$\text{AM} = \text{GM}$$

Hence,

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$$
$$x = 3,\ y = 4, z = 5$$

Thus the required value is calculated as,

$$x^3 + y^3 + z^3 = 27 + 64 + 125$$
$$= 216$$

69. Consider the given equation and simplify:

$$\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)=\sum_{r=1}^{15} r^2\left(\frac{15-r+1}{r}\right)$$

$$=\sum_{r=1}^{15} r(16-r)$$

$$=16\left(\frac{15\times16}{2}\right)-\frac{16\times15\times31}{6}$$

$$=\frac{15\times16}{6}(17)$$

$$=680$$

70. The given function $L=\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}$ is in the from of 1^{∞}.

So simplify the function.

$$L=e^{\lim_{x\to\infty}\left(\frac{a}{x}-\frac{4}{x^2}\right)(2x)}$$

$$=e^{\lim_{x\to\infty}\left(\frac{2(ax-4)}{x}\right)}$$

$$=e^{2a}$$

Now apply the given condition of question:

$$e^{2a}=e^3$$

$$a=\frac{3}{2}$$

71. Consider the given function,

$$f(x)=\begin{cases}-x, & x<1\\ a+\cos^{-1}(x+b), & 1\le x\le 2\end{cases}$$

Condition for the function if it is differentiable is,

$$\underbrace{f'(1^-)}_{\text{LHL}}=\underbrace{f'(1^+)}_{\text{RHL}}$$

Calculate the right hand limit (RHL),

$$f'(1^+)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{a+\cos^{-1}(1+h+b)-a-\cos^{-1}(1+b)}{h}$$

$$=\lim_{h\to 0}\frac{\cos^{-1}((1+b)+h)-\cos^{-1}(1+b)}{h}$$

Apply L-hospital's rule,

$$f'(1^+)=\lim_{h\to 0}\frac{\frac{-1}{\sqrt{1-((1+b)+h)^2}}-0}{1}$$

$$=\frac{-1}{\sqrt{1-(1+b)^2}} \qquad \ldots\ldots (1)$$

Calculate the left hand limit (LHL),

$$f'\left(1^-\right)=\lim_{h\to 0}\frac{f\left(1+h\right)-\left(1\right)}{h}$$

$$=\lim_{h\to 0}\frac{-\left(1+h\right)+1}{h}$$

$$=-1 \qquad \ldots\ldots (2)$$

Therefore, from the equation (1) and (2),

$$f\left(1^+\right)=f\left(1^-\right)$$

$$-1=\frac{-1}{\sqrt{1-\left(1+b\right)^2}}$$

$$1-\left(1+b\right)^2=1$$

$$b=-1$$

Also,

$$f\left(1^-\right)=f\left(1^+\right)$$

$$-1=a+\cos^{-1}\left(1+b\right)$$

Substitute the value of b in the above expression,

$$-1=a+\cos^{-1}0$$

$$-1=a+\frac{\pi}{2}$$

$$a=-1-\frac{\pi}{2}$$

Hence, the value of $\frac{a}{b}$ is,

$$\frac{a}{b}=\frac{\pi+2}{2}$$

72. Consider the equation of the given curves,

$$x=4t^2+3$$
$$y=8t^3-1$$

Differentiate the curves with respect to t,

$$\frac{dx}{dt}=8t+0$$
$$=8t$$

And,

$$\frac{dy}{dt}=24t^2-0$$
$$=24t^2$$

Calculate the value of $\frac{dy}{dx}$,

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$$
$$=\frac{24t^2}{8t}$$
$$=3t$$

The equation of the tangent at point $P(x_1, y_1)$ is given by,

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

Here, $(x_1, y_1) \equiv (4t^2 + 3, 8t^3 - 1)$.

Substitute the value in the above equation of the tangent,

$$y - 8t^3 + 1 = 3t(x - 4t^2 - 3)$$

Consider the point Q $(4t_1^2 + 3, 8t_1^3 - 1)$ will satisfy the above equation of the tangent at point P.

Therefore,

$$8t_1^3 - 1 - 8t^3 + 1 = 3t(4t_1^2 + 3 - 4t^2 - 3)$$

$$8(t_1 - t)(t_1^2 + t_1 t + t^2) = 3t \cdot 4(t_1 - t)(t_1 + t)$$

$$2t_1^2 + 2t_1 t - t^2 = 0$$

$$(t_1 - t)(2t_1 + t) = 0$$

Simplify the above equation to find the value of t

$$t_1 = -\frac{t}{2}$$

Hence, the point Q is $(t^2 + 3, -t^3 - 1)$.

73. Let the point at minimum distance from the origin O be $(h, h^2 - 4)$.

Hence,

$$OP^2 = h^2 + (h^2 - 4)^2$$

Differentiate the above equation and equate to zero.

$$\frac{d\left(OP^2\right)}{dh}=0$$

$$2h+2\left(h^2-4\right)2h=0$$

$$h=\pm\sqrt{\frac{7}{2}},0$$

Again differentiate the above equation:

$$\left(\frac{d^2\left(OP^2\right)}{dh^2}\right)_{h=\pm\sqrt{\frac{7}{2}}}>0$$

Then OP is minimum at $h=\pm\sqrt{\frac{7}{2}}$

Calculate the minimum distance,

$$OP_{\min}=\sqrt{\frac{7}{2}+\left(\frac{7}{2}-4\right)^2}$$
$$=\frac{\sqrt{15}}{2}$$

74. Consider the given integral.

$$\int\frac{dx}{\cos^3\sqrt{2\sin 2x}}=\frac{1}{2}\int\frac{\left(\tan^2 x+1\right)\sec^2 x}{\left(\tan x\right)^{\frac{1}{2}}}dx$$

Substitute $\tan x = t$.

$$I = \frac{1}{2}\int t^{\frac{3}{2}} dt + \frac{1}{2}\int t^{\frac{-1}{2}} dt$$

$$= \frac{t^{\frac{5}{2}}}{5} + t^{\frac{1}{2}} + c$$

$$= \frac{(\tan x)^{\frac{5}{2}}}{5} + (\tan x)^{1/2}$$

Equate the above equation with the given equation:

$$A = \frac{1}{2}$$

$$B = \frac{5}{2}$$

$$C = \frac{1}{5}$$

Calculate the value of $A + B + C$,

$$A + B + C = \frac{1}{2} + \frac{5}{2} + \frac{1}{5}$$

$$= \frac{16}{5}$$

75. Consider the given equation is,

$$2\int_0^1 \tan^{-1} x \, dx = \int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx$$

Integrate and find the value of the integral,

$$2\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx = 2\int_0^1\left(\frac{\pi}{2}-\cot^{-1}\left(1-x+x^2\right)dx\right)$$

$$= \left.\frac{\pi x}{2}\right|_0^1 - 2\int_0^1 \tan^{-1} x dx$$

$$= \frac{\pi}{2} - 2\left(\frac{\pi}{4}-\frac{1}{2}\ln 2\right)$$

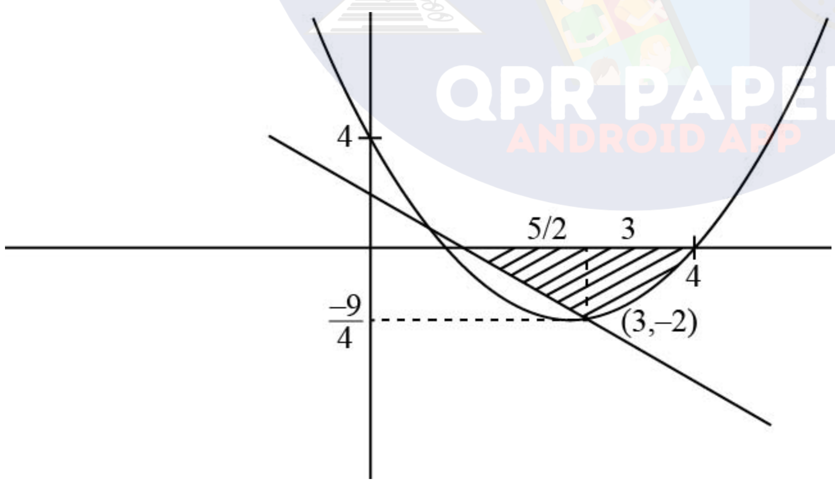
$$= \ln 2$$

76. Consider the given area:

$$\mathrm{A} = \{(x, y) \mid y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\}$$

Here $y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0$

Consider the required plot:



Calculate the required area is,

$$\text{Area} = \frac{1}{2}\times 2\times 2+\int_{3}^{4}\left(5x-x^{2}-4\right)dx$$

$$= 2+\left(\frac{5x^{2}}{2}-\frac{x^{3}}{3}-4x\right)_{3}^{4}$$

$$= 2+\frac{5}{2}(16-9)-\frac{1}{3}(64-27)-4(4-3)$$

$$= \frac{19}{6}$$

77. Consider the given function,

$$\lim_{t\to x}\frac{t^{2}\cdot f(x)-x^{2}f(t)}{t-x}=1$$

Apply the L-hospital's rule in the above equation,

$$\lim_{t\to x}\frac{2t\cdot f(x)-x^{2}f'(t)}{1-x}=1$$

$$\frac{2xf(x)-x^{2}f'(x)}{1-0}=1$$

$$-x^{2}f'(x)=1-2xf(x)$$

$$f'(x)=\frac{2xf(x)-1}{x^{2}}$$

Replace, $f'(x)$ with $\frac{dy}{dx}$,

$$\frac{dy}{dx}=\frac{2xy}{x^2}-\frac{1}{x^2}$$

$$\frac{dy}{dx}-y\left(\frac{2}{x}\right)=-\frac{1}{x^2}$$

Compare the above equation with given equation,

$$\frac{dy}{dx}+Px=Q$$

Now, calculate the value of the integrating factor,

$$\text{I.F.}=e^{\int Pdx}$$

Substitute the value in the above IF,

$$\text{I.F.}=e^{-\int\frac{2}{x}dx}$$
$$=e^{-2\ln x}$$
$$=\frac{1}{x^2}$$

The solution of the differential equation is,

$$y(P)=\int Q\times(I.F.)dx$$

Substitute the value,

$$y\left(\frac{1}{x^2}\right)=\int-\frac{1}{x^4}dx$$
$$\frac{y}{x^2}=\frac{1}{3x^2}+\text{c}$$

Substitute the value $x=1,\ y=1$ in the above equation,

$$\frac{1}{1}=\frac{1}{3}+c$$

$$c=1-\frac{1}{3}$$

$$=\frac{2}{3}$$

Substitute the value in the equation,

$$\frac{y}{x^2}=\frac{1}{3x^2}+\frac{2}{3}$$

$$f(x)=\frac{1}{3x}+\frac{2x^2}{3}$$

Calculate the value of $f\left(\frac{3}{2}\right)$,

$$f(x)=\frac{1}{3x}+\frac{2x^2}{3}$$

$$=\frac{1}{3\times\left(\frac{3}{2}\right)}+\frac{2}{3}\times\left(\frac{3}{2}\right)^2$$

$$f\left(\frac{3}{2}\right)=\frac{31}{18}$$

78. Consider the given equations:

$$\frac{x}{3}+\frac{y}{4}=1 \qquad \text{..... (1)}$$

$$4x+3y=12$$

$$\frac{x}{4}+\frac{y}{3}=1 \qquad \text{...... (2)}$$

$$3x+4y=12$$

Equation of line passing through the intersection is given as:

$$4x+3y-12+\lambda(3x+4y-12)=0$$

Let A and B are the intersection point.

$$\text{A}=C\left(\frac{12(1+\lambda)}{4+3\lambda},0\right)$$

$$\text{B}=\left(0,\frac{12(1+\lambda)}{3+4\lambda}\right)$$

Then ,midpoint $P(h,k)$ is given by:

$$h=\frac{6(1+\lambda)}{4+3\lambda} \qquad \text{...... (3)}$$

$$k=\frac{6(1+\lambda)}{3+4\lambda} \qquad \text{...... (4)}$$

From equation (3) and (4),

$$\lambda=\frac{3k-4h}{3h-4k}$$

Substitute the value of λ in equation (1).

$$7xy = 6(x+y)$$

79. Consider the given the equation of the line,

$$x - y = 4$$

$$y = x - 4 \qquad \ldots\ldots (1)$$

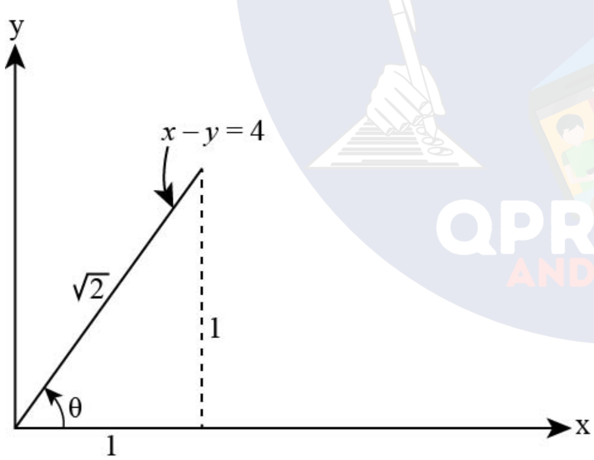
Compare the above equation with $y = mx + c$ the value of the slope of the line is,

$$m = 1$$

Or,

$$\tan\theta = 1$$

Here θ is the angle between x and line as shown in the figure.



Therefore,

In the first quadrant the value of $\cos\theta$ is,

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

In the third quadrant the value of $\cos\theta$,

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

$$\sin\theta = -\frac{1}{\sqrt{2}}$$

Then the new point Q is,

$$Q \equiv \left(x + 2\sqrt{3}\left(\cos\theta\right)\right), y + 2\sqrt{3}\left(\sin\theta\right)$$

$$\equiv \left(2 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right), 1 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right)\right)$$

$$\equiv \left(2 - \sqrt{6}, 1 - \sqrt{6}\right)$$

Hence, the Equation of required line is

$$x + y = 3 - 2\sqrt{6}$$

Slope of $x - y = 4$ is given as:

$$\tan\theta = 1 \text{ then } \left(\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}\right) \text{ or}$$

$$\left(\sin\theta = -\frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}\right)$$

Calculate the quadrant of Q.

$$\left(2+2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right),1+2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right)\right)$$

$$Q\left(2-\sqrt{6},1-\sqrt{6}\right)$$

Equation of required line is $x+y=3-2\sqrt{6}$

80. Consider the equation of the circle,

$$(x-h)^2+(y-k)^2=r^2 \qquad \ldots\ldots (1)$$

Here, the centre of the circle is (h,k) and radius is r.

Required circle is given as:

$$x^2+(y-2)^2+\lambda x=0$$

This passes through $(-2,4)$, Hence,

$$4+4-2\lambda=0$$

$$\lambda=4$$

Substitute the value of the λ in the equation of the circle,

$$x^2+(y-2)^2+4x=0$$

$$x^2+y^2-4y+4x+4+4=4$$

$$\left(x^2+4x+4\right)+\left(y^2-4y+4\right)=4$$

$$(x+2)^2+(y-2)^2=2^2$$

Compare this equation with equation (1), The centre of the circle is $(-2, 2)$ and radius is 2.

The centre $(-2,2)$ which satisfy the equation,

$$2x-3y+10=0$$

Thus, the option (2) is correct.

81. Consider the given equation,

$$9e^2-18e+5=0$$

$$e=\frac{5}{3}$$

The formula to calculate eccentricity is,

$$e=\frac{\sqrt{(a^2+b^2)}}{a}$$

From above formula,

$$1+\frac{b^2}{a^2}=e^2$$

$$1+\frac{b^2}{a^2}=\frac{25}{9} \qquad \text{...... (1)}$$

The distance between foci and directrix is calculated as,

$$\left(ae-\frac{a}{e}\right)=5-\frac{9}{5}$$

$$a\left(\frac{5}{3}-\frac{3}{5}\right)=\frac{16}{5}$$

$$a=3$$

From equation (1),

$$1+\frac{b^2}{9}=\frac{25}{9}$$

$$b^2=16$$

So,

$$a^2-b^2=9-16$$

$$=-7$$

82. The general equation of the ellipse is given by,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

The equation of the ellipse is given as,

$$\frac{x^2}{27}+\frac{y^2}{3}=1$$

The point P on the ellipse is given by,

$$P\equiv\left(a\cos\theta, b\sin\theta\right)$$

Substitute the value of the value of a and b in the point P,

$P \equiv \left(3\sqrt{3}\cos\theta, \sqrt{3}\sin\theta\right)]\backslash$

At the point P$\left(3\sqrt{3}\cos\theta, \sqrt{3}\sin\theta\right)$

The equation of the tangent is given by,

$$\frac{x\cos\theta}{a^2} + \frac{y\sin\theta}{b^2} = 1$$

Substitute the value in the above equation,

$$\frac{x}{3\sqrt{3}}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$$

The value of a is given by,

$$a = \left(3\sqrt{3}\sec\theta, 0\right)$$

The value of b is given by,

$$b = \left(0, \text{cosec}\,\theta\right)$$

Area of triangle is caclulated as:

$$\begin{aligned}\text{Area}\left(\Delta\text{OAB}\right) &= \frac{1}{2} \times \text{OA} \times \text{OB} \\ &= \frac{1}{2}\left(3\sqrt{3}\sec\theta \cdot \sqrt{3}\cos ec\theta\right) \\ &= \frac{9}{2\sin\theta\cos\theta} \\ &= \frac{9}{\sin\theta}\end{aligned}$$

Hence, the minimum area of ΔOAB is calculated as,

$$\text{Area}_{\text{Min}} = \frac{9}{\sin\theta}$$

$$= \frac{9}{1} \quad [\because \text{maximum value of } \sin\theta \text{ is } 1]$$

$$= 9$$

83. Consider the given equations of line,

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$$

$$\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

Shortest distance d between the lines is given by,

$$d = \left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1 b_2 - a_2 b_1)^2 + (b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2}} \right|$$

Substitute the value in the above expression,

$$d=\left|\frac{\begin{vmatrix}-2-0 & 4-0 & 5-0\\ 2 & 2 & 1\\ -1 & 8 & 4\end{vmatrix}}{\sqrt{\left(16-(-2)\right)^2+(8-8)^2+(-1-8)^2}}\right|$$

$$=\left|\frac{-2(8-8)-4(8+1)+5(16+2)}{\sqrt{324+81}}\right|$$

$$=2.7$$

Hence, this value is lies in $(2,3]$.

84. Consider the given equation of planes,

$$x-y+2z=3$$

$$2x-2y+z+12=0$$

And it passes through $(1,2,2)$ is,

$$\begin{vmatrix}x-1 & y-2 & z-2\\ 1 & -1 & 2\\ 2 & -2 & 1\end{vmatrix}=0$$

Solve the matrics,

$$(x-1)(-1+4)-(y-2)(1-4)+(z-2)(-2+2)=0$$

$$3(x-1)+3(y-2)=0$$

$$x+y=3$$

…… (1)

Then the distance of plane $x+y-3=0$ from $(1,-2,4)$ is calculated as,

$$d_{(1,-2,4)}=\left|\frac{x+y-3}{\sqrt{(1)^2+(1)^2}}\right|$$

$$d_{(1,-2,4)}=\left|\frac{1-2-3}{\sqrt{(1)^2+(1)^2}}\right|$$

$$=2\sqrt{2}$$

85. The given value of the position vector is,

$\vec{A}=3\hat{i}+\hat{j}-\hat{k}$

$\vec{B}=-\hat{i}+3\hat{j}+p\hat{k}$

$\vec{C}=5\hat{i}+q\hat{j}-4\hat{k}$

Dot product of the position vector of A and B is,

$\vec{A}.\vec{B}=-4\hat{i}+2\hat{j}+(p+1)\hat{k}$

Dot product of the position vector of A and C is,

$\vec{A}.\vec{C} = 2\hat{i} + (q-1)\hat{j} - 3\hat{k}$

The relation for right angle triangle is,

$$\left(\vec{A}.\vec{B}\right).\left(\vec{A}.\vec{C}\right) = 0$$

$$-8 + 2(9-1) - 3(p+1) = 0$$

$$-3p + 2q - 13 = 0$$

$$3p - 2q + 13 = 0$$

Hence, the point (p, q) lies on a line:

$3x - 2y + 13 = 0$

$$y = \left(\frac{3}{2}\right)x + \frac{13}{2}$$

Hence, the slope is,

$\text{slope} = \frac{3}{2}$

86. The formula of the mean of the series is given by,

$\overline{x} = \sum_{i=1}^{n} \left(\frac{x_i}{n}\right)$

The given series in A.P. is,

$1, 1+d, 1+2d, ..., 1+100d$

Apply the formula of A.P to calculate the number of terms in this series,

$1+100d=1+(n-1)d$

Compare both sides

$n-1=100$

$n=101$

The sum of the 101 terms is calculated as,

$$x_i=\frac{101}{2}\left(2+(101-1)d\right)\quad\left[\because S_{\text{A.P.}}=\frac{n}{2}\left(2a+(n-1)d\right)\right]$$

$$=101(50d+1)$$

Therefore, mean is given as,

$$\overline{x}=\frac{101(50d+1)}{101}$$

$$=1+50d$$

Sum of deviation about mean is given as:

$$50d+49d+\ldots\ldots\ldots\ldots\ldots+d+0+d+\ldots\ldots.+50d=50\times51d$$

Hence, the mean deviation from mean is

$$\sigma=\frac{50\times51d}{101}$$

$$255=\frac{50\times51d}{101}$$

$$d=10.1$$

Simplify further and find the value of d:

$$d = \frac{255 \times 101}{2550}$$
$$= 10.1$$

87. The given probability is :

$$\mathrm{P}(\mathrm{A}) = \frac{2}{5}$$

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{3}{20}$$

Hence,

$$\mathrm{P}\left(\mathrm{A} \mid (\mathrm{A}' \cup \mathrm{B}')\right) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{A}' \cup \mathrm{B}')}{\mathrm{P}(\mathrm{A}' \cap \mathrm{B}')}$$

$$= \frac{\mathrm{P}\left((\mathrm{A} \cap \mathrm{A}') \cup (\mathrm{A} \cap \mathrm{B}')\right)}{\mathrm{P}(\mathrm{A} \cap \mathrm{B})'}$$

$$= \frac{\mathrm{P}\left(\phi \cup (\mathrm{A} \cap \mathrm{B}')\right)}{1 - P(\mathrm{A} \cap \mathrm{B})}$$

$$= \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B}')}{1 - \frac{3}{20}}$$

Further simplify the above equation:

$$P\left(A \mid \left(A' \cup B'\right)\right) = \frac{P(A) - P(A \cap B)}{\frac{17}{20}}$$

$$= \frac{\frac{2}{5} - \frac{3}{20}}{\frac{17}{50}}$$

$$= \frac{5}{17}$$

88. Consider the given expression,

$$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$$

Simplify the above expression,

$$\left|\sqrt{2\sin^4 x + 18\cos^2 x}\right| = 1 + \left|\sqrt{2\cos^4 x + 18\sin^2 x}\right|$$

Squaring both sides of the given equation

$$2\sin^4 x + 18\cos^2 x = 1 + 2\cos^4 x + 18\sin^2 x$$
$$+2\sqrt{2\cos^4 x + 18\sin^2 x}$$

$$2\left(\sin^4 x - \cos^4 x\right) + 18\left(\cos^2 x - \sin^2 x\right) = 1 + 2\sqrt{2\cos^4 x + 18\sin^2 x}$$

$$2\left(\sin^4 x - \cos^4 x\right) + 18\left(\cos^2 x - \sin^2 x\right) = \sin^2 x + \cos^2 x$$
$$+2\sqrt{2\cos^4 x + 18\sin^2 x}$$

$$2\left(\frac{1-\cos 2x}{2}\right)^2 - 2\left(\frac{1+\cos 2x}{2}\right)^2 + 18\cos 2x - 1 = 2\sqrt{2\left(\frac{1+\cos 2x}{2}\right)^2 + 9\left(1-\cos 2x\right)}$$

Further simplify the above equation:

$$\begin{pmatrix} 1+\cos^2 2x-2\cos 2x \\ -1-\cos^2 2x-2\cos 2x \end{pmatrix}+36\cos 2x-2=4\sqrt{2\left(\frac{1+\cos 2x}{2}\right)^2 +9(1-\cos 2x)}$$

$$-4\cos 2x+36\cos 2x=4\sqrt{2\left(\frac{1+\cos 2x}{2}\right)^2 +9(1-\cos 2x)}$$

$$8\cos 2x=\sqrt{2\left(\frac{1+\cos 2x}{2}\right)^2 +9(1-\cos 2x)}$$

$$64\cos^2 2x=2\left(\frac{1+\cos 2x}{2}\right)^2+9(1-\cos 2x)$$

Further simplify the equation:

$$64\cos^2 2x-9+9\cos 2x-\frac{1}{2}\left(1+\cos^2 2x+2\cos 2x\right)=0$$

$$-127\cos^2 2x-19+16\cos 2x=0$$

$$\cos 2x=\pm\sqrt{\frac{37}{254}}\in[-1,1]$$

Hence, clearly there are 8 solutions.

89. Consider the given equation,

$$4+\frac{1}{2}\sin^2 2x-2\cos^4 x$$

Simplify the above equation:

$$\begin{aligned}4+\frac{1}{2}\sin^2 2x-2\cos^4 x &= 4+\frac{1}{2}\sin^2 x-\frac{1}{2}\left(2\cos^2 x\right)^2\\ &= 4+\frac{1}{2}\sin^2 2x-\frac{1}{2}(1+\cos 2x)^2\\ &= -\cos^2 2x-\cos 2x+4\\ &= -\left[\cos^2 x+\cos 2x-4\right]\end{aligned}$$

Further simplify the above equation:

$$4+\frac{1}{2}\sin^2 2x-2\cos^4 x=\frac{17}{4}-\left(\cos 2x+\frac{1}{2}\right)^2$$

M denotes for maximum value and m denotes the minimum value:

The $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$ is maximum when the value of the

$\left(\cos 2x+\frac{1}{2}\right)$ is minimum,

$$M=\frac{17}{4}$$

And minimum value is,

$$m=2$$

Hence, calculate the difference between M and m,

$$M - m = \frac{17}{4} - 2$$

$$= \frac{9}{4}$$

90. For the statement P,

Contrapositive of P is, "If 7 is not divisible by 2, then it is not an odd number." It is false.

For the statement Q,

Contrapositive of Q is "If 7 is not an odd number, then 7 is not a prime number". It is true.

Hence, the ordered pair of the $(V_1, V_2) \equiv (F, T)$.