JEE MAIN 2020 4th SEPT SHIFT 2

Physics

Question 1. Two disc made of same material and same thickness having radius R and αR. Their moment of inertia about their own axis are in ratio 1 : 16. Calculate the value of α

(1) 2

(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{4}$

Ans. (1)

Sol. Moment of inertia of disc is given by

$$I = \frac{MR^2}{2} = \frac{\left[\rho\left(\pi R^2\right)t\right]R^2}{2}$$

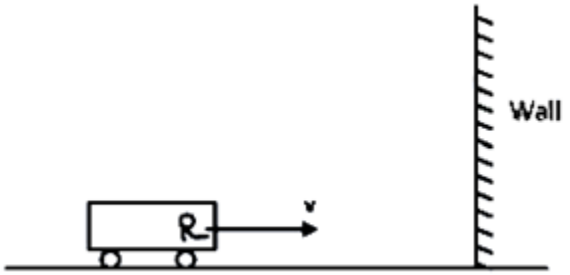
$$I \propto R^4$$

$$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4$$

$$\frac{16}{1} = \alpha^4$$

$$\alpha = 2$$

Question 2. Bus moving with speed v towards a stationary wall. It produces sound of frequency f = 420 Hz. The heard frequency of reflected sound from wall by driver is 490 Hz. Calculate the speed v of bus. The velocity of sound in air is 330 m/s.



(1) 61 Km/s

(2) 71 Km/s

(3) 81 Km/s

(4) 91 Km/s

Ans. (4)

Sol. Frequency appeared at wall

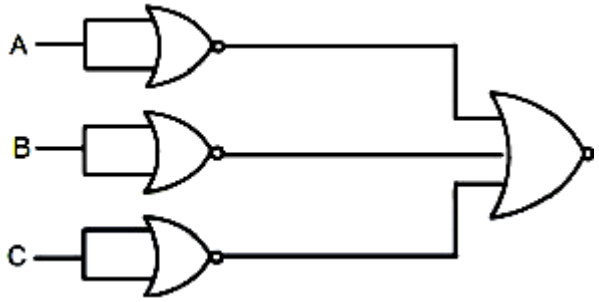
$$f_w = \frac{330}{330 - v}.f \quad \ldots (1)$$

$$f' = \frac{330 + v}{330}.f_w = \frac{330 + v}{330 - v}.f$$

$$490 = \frac{330 + v}{330 - v}.420$$

$$v = \frac{330 \times 7}{91} \approx 25.38 \text{ m/s} = 91 \text{ Km/s}$$

Question 3. The given circuit behaves like a following single gate



(1) OR

(2) AND

(3) NAND

(4) NOR

Ans. (2)

Sol. 

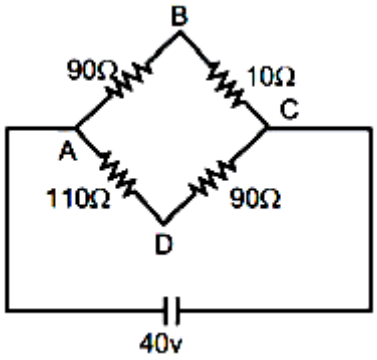
Behaves like a not gate so Boolean equation will be

$$y = \overline{\overline{A} + \overline{B} + \overline{C}}$$

$$y = A.\ B.\ C$$

Whole arrangement behaves like a AND gate

Question 4.



In the given circuit calculate the potential difference between points A and B.

(1) 12 V

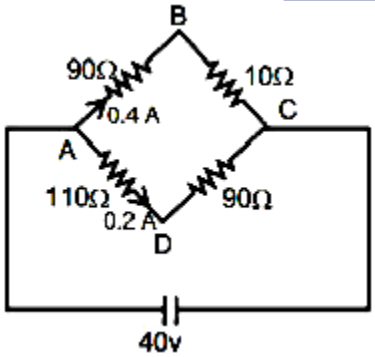
(2) 24 V

(3) 36 V

(4) 48 V

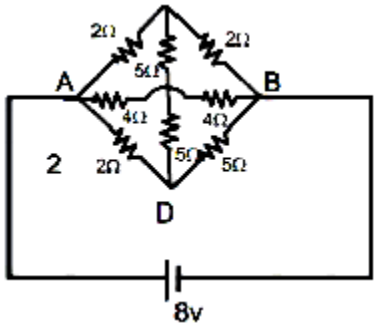
Ans. (3)

Sol.



From ohm's law VAB = 90 × 0.4 = 36 V

Question 5. Find current through 4Ω resistance



(1) 1 Amp

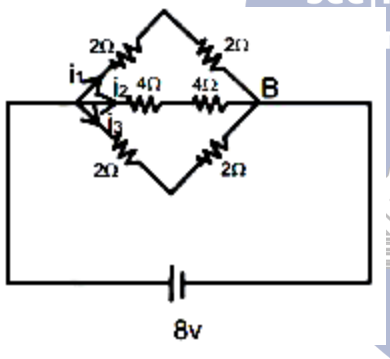
(2) 2 Amp

(3) 3 Amp

(4) 4 Amp

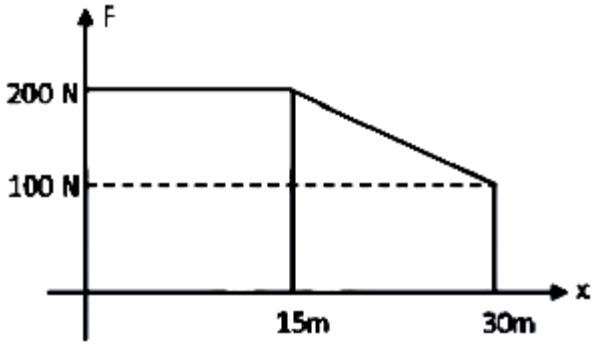
Ans. (1)

Sol.



$$i_2 = \frac{8}{4+4} = 1 \text{ Amp}$$

Question 6. Force on a particle varies with position (x) of particle as shown, calculate work done by force from x = 0 to x = 30 m



(1) 5250 J

(2) 4250 J

(3) 7500 J

(4) 3750 J

Ans. (1)

Sol. $W = \text{area} = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15$

$= 3000 + 2250$

$W = 5250\ J$

Question 7. A capacitor of capacitance C0 is charged to potential V0. Now it is connected to another uncharged capacitor of capacitance $\frac{C_0}{2}$. Calculate the heat loss in this process.

(1) $\frac{1}{2}C_0V_0^2$

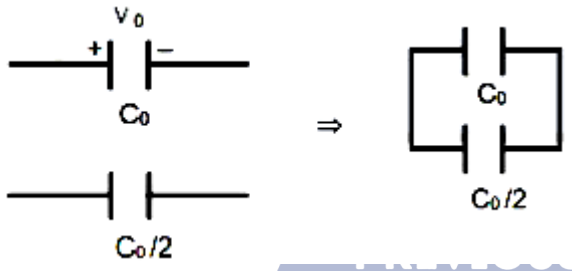
(2) $\frac{1}{3}C_0V_0^2$

(3) $\frac{1}{6}C_0V_0^2$

(4) $\frac{1}{8}C_0V_0^2$

Ans. (3)

Sol.



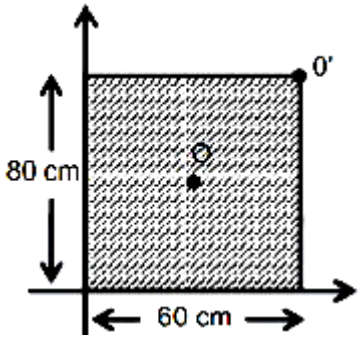
heat loss

$$H=\frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$$

$$=\frac{C\times\frac{C}{2}}{2\left(C+\frac{C}{2}\right)}(V_0-0)^2=\frac{C}{6}V_0^2$$

$$H=\frac{1}{6}C_0V_0^2$$

Question 8. Find the ratio of moment of inertia about axis perpendicular to rectangular plate passing through O' & O



(1) $\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{1}{4}$

(4) $\frac{1}{8}$

Ans. (3)

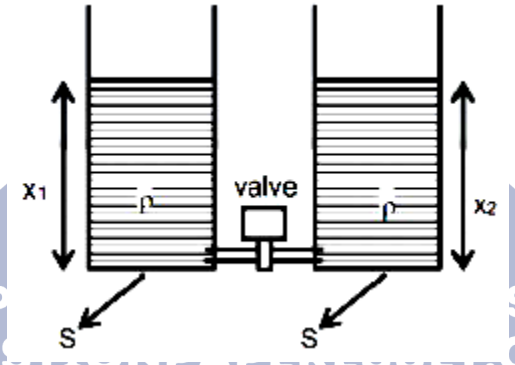
Sol.

$$\frac{I_o}{I_{o'}} = \frac{\frac{M}{12}\left(a^2+b^2\right)}{\frac{M}{12}\left(a^2+b^2\right)+m\left(\frac{a^2}{4}+\frac{b^2}{4}\right)}$$

$$\frac{\frac{M}{12}\left(a^2+b^2\right)}{\frac{M}{3}\left(a^2+b^2\right)} = 4$$

$$\frac{I_{o'}}{I_o} = \frac{1}{4}$$

Question 9. Find the loss in gravitational potential energy of cylinder when valve is opened and level of liquid in both cylinder become same



(1) $\frac{\rho Ag\left(x_1 - x_2\right)^2}{4}$

(2) $\frac{\rho Ag\left(x_1 + x_2\right)^2}{4}$

(3) $\frac{\rho Ag\left(x_1^2 - x_2^2\right)}{4}$

(4) $\frac{\rho Ag\left(x_1^2 + x_2^2\right)}{4}$

Ans. (1)

Sol. Initial height of liquid in containers of same cross section are x1 and x2 respectively. Now value is opened find

loss in potential energy when water level be become same

loss in PE = Ui - Uf

$$= \left[\rho(A) x_1 \frac{x_1}{2} + \rho A x_2 \frac{x_2}{2} \right] g$$

$$- \left[\rho A \left(\frac{x_1 + x_2}{2} \right) \times \left(\frac{x_1 + x_2}{4} \right) \times 2 \right] g$$

$$= \rho A g \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \frac{(x_1 + x_1)^2}{4} \right] = \frac{\rho A g (x_1 - x_2)^2}{4}$$

Question 10. A coil has moment of inertia 0.8 kg/m^2 released in uniform magnetic field 4T when there is 60° angle between magnetic field and magnetic moment of coil. Magnetic moment of coil is 20 A-m^2. Find the angular speed of coil when it passes through stable equilibrium.

(1) 20 π rad / s^{-1}

(2) 20 rad / s^{-1}

(3) 10 π rad / s^{-1}

(4) 10 rad / s^{-1}

Ans. (4)

Sol. From energy conservation

$$\frac{1}{2} I\omega^2 = U_{in} - U_f$$

$= -MB\cos 60° - (-MB)$

$$\frac{MB}{2} = \frac{1}{2}I\omega^2$$

$$\frac{20\times 4}{2} = \frac{1}{2}(0.8)\omega^2$$

$$100 = \omega^2$$

$$\omega = 10 \text{ rad/s}$$

Question 11. A charged particle of charge q released in electric field $E = E_0(1 - ax^2)$ from origin. Find position where its kinetic energy again becomes zero.

(1) $\sqrt{\frac{1}{a}}$

(2) $\sqrt{\frac{2}{a}}$

(3) $\sqrt{\frac{3}{a}}$

(4) $2\sqrt{\frac{1}{a}}$

Ans. (3)

Sol. $dW_{ex} = \Delta K \quad K_f - K_i = 0$

$$\int_0^x qEdx = 0$$

$$q\int_0^x E_0\left(1-ax^2\right)dx = 0$$

$$qE_0\int_0^x\left(1-ax^2\right)dx = 0$$

$$x-\frac{ax^3}{3}=0$$

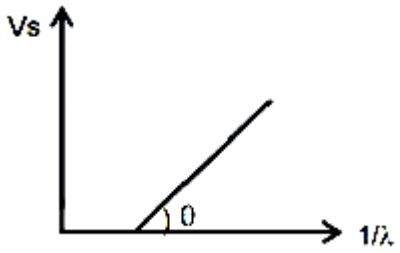
$$1-\frac{ax^2}{3}=0$$

$$\frac{ax^2}{3}=1$$

$$x^2=\frac{3}{a}$$

$$x=\pm\sqrt{\frac{3}{a}}$$

Question 12. A light is incident on a metallic surface. Graph between stopping potential Vs and $\frac{1}{\lambda}$ is as shown in figure. When intensity of light is increase at given frequency then:



(1) Graph does not change

(2) Graph steeper

(3) VS intercept change

(4) Graph gets narrower

Ans. (1)

Sol. $eVs = hv - w$

$$Vs = \frac{hv}{e} - \frac{W}{e}$$

Frequency and work function are constant therefore graph does not change.

Question 13. A ball is thrown with velocity v_0 from ground in vertical upward direction. If particle experiences resistance force mkv^2. Where v is the speed of particle, m mass of the particle and k is a positive constant. Find maximum height reached.

(1) $\frac{1}{2K}\ell n\left(\frac{g+kv_0^2}{g}\right)$

(2) $\frac{1}{3K}\ell n\left(\frac{g+kv_0^2}{g}\right)$

(3) $\frac{2}{3K}\ell n\left(\frac{g+kv_0^2}{g}\right)$

(4) $\frac{1}{K}\ell n\left(\frac{g+kv_0^2}{g}\right)$

Ans. (1)

Sol. $F_{net} = ma$

$$-mg - mkv^2 = mv\frac{dv}{ds}$$

$$v\frac{dv}{ds} = -g - kv^2$$

$$-\int_{v_0}^{0} \frac{vdv}{g + kv^2} = \int_{0}^{h_{max}} ds = h_{max}$$

$$h_{max} = \frac{1}{2k}\ell n\left(\frac{g + kv_0^2}{g}\right)$$

Question 14. $\lambda = 6000 \times 10^{-10}$ m and width : 0.6×10^{-4}m. Find highest order of minima on both side of central maxima

(1) 10

(2) 20

(3) 100

(4) 200

Ans. (3)

Sol. Light of wavelength 6000×10^{-10}m passes through a single slit of width 0.6×10^{-4}m. Find height of highest order of minima on both side central maxima

for minima

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} < 1$$

$$n \le \frac{d}{\lambda}$$

$$n < \frac{0.6\times10^{-4}}{6000\times10^{-10}}$$

$$n < 100$$

Question 15. Maximum wavelength of Lyman series photon for H is λ then minimum wavelength of balmer series photon for He+ atm

(1) $\frac{\lambda}{4}$

(2) $\frac{3\lambda}{4}$

(3) $\frac{\lambda}{4}$

(4) $\frac{2\lambda}{3}$

Ans. (2)

Sol. $\frac{1}{\lambda He^{+}} = R(4)\left(\frac{1}{4}-\frac{1}{\infty}\right) = R$

$$\frac{1}{\lambda He^{+}} = \frac{1}{R}$$

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right) \text{ given}$$

$$\frac{1}{\lambda} = \frac{3R}{4}$$

$$R = \frac{4}{3\lambda}$$

$$\therefore \lambda_{He^+} = \frac{3\lambda}{4}$$

Question 16. Electric field in EM waves is $E = E_0\left(\hat{i} + \hat{j}\right).\sin\left(kz - \omega t\right)$, then equation of magnetic field is:

(1) $B = B_0\left(-\hat{i} + \hat{j}\right)\sin\left(kz - \omega t\right)$

(2) $B = B_C\left(\hat{i} + \hat{j}\right)\sin\left(kz - \omega t\right)$

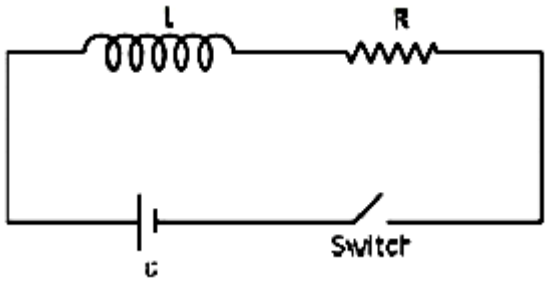
(3) $B = B_C\left(\hat{j} + \hat{k}\right)\sin\left(kz - \omega t\right)$

(4) $B = B_0\left(\hat{i} + \hat{j}\right)\sin\left(kz - \omega t\right)$

Ans. (1)

Sol. $\vec{E} \times \vec{B} \parallel \vec{C}$

Question 17. The circuit is switched on at t = 0, Find the time when energy stored in inductor becomes $\frac{1}{n}$ times of maximum energy stored in it:



(1) $\frac{L}{R}\ln\frac{\sqrt{n}}{\sqrt{n+1}}$

(2) $\frac{L}{R}\ln\frac{\sqrt{n}}{\sqrt{n-1}}$

(3) $\frac{L}{R}\ln\frac{\sqrt{n+1}}{\sqrt{n}}$

(4) $\frac{L}{R}\ln\frac{\sqrt{n-1}}{\sqrt{n}}$

Ans. (2)

Sol. Potential energy stored in inductor is given by

$$U = \frac{1}{2}LI^2$$

$$U \propto I^2$$

$$\frac{U}{U_0} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{1}{n} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{I}{I_0} = 1 - e^{-RT/L} = \frac{1}{\sqrt{n}}$$

$$t = \frac{L}{R} \ln \frac{\sqrt{n}}{\sqrt{n} - 1}$$

Question 18. Intensity of magnetization is 4 (unit) at temperature 6K and B = 0.4 T. What is the intensity of magnetization at temperature 24 K in B = 0.3 T

(1) 0.75

(2) 0.25

(3) 0.5

(4) 1

Ans. (1)

Sol. Magnetization = 4 T = 6k, B = 0.4 T

(Paramagnetic substance) T = 24k, B = 0.3 T

$$M = \frac{CB_{ex}}{T}$$

$$\frac{4}{M} = \frac{0.4/6}{0.3/24} = 0.75$$

Question 19. Match the following

I	Adiabatic	(A) $\Delta U = 0$
II	Isothermal	(B) $\Delta W = 0$
III	Isobaric	(C) $\Delta Q = 0$
IV	Isochoric	(D) $\Delta U \neq 0$, $\Delta Q \neq 0$, $\Delta W \neq 0$

(1) I $\rightarrow$ A II $\rightarrow$ C III $\rightarrow$ D IV $\rightarrow$ B

(2) I $\rightarrow$ D II $\rightarrow$ B III $\rightarrow$ C IV $\rightarrow$ A

(3) I $\rightarrow$ C II $\rightarrow$ A III $\rightarrow$ D IV $\rightarrow$ B

(4) I $\rightarrow$ B II $\rightarrow$ D III $\rightarrow$ C IV $\rightarrow$ A

Ans. (3)

Question 20. A Satellite is revolving around the earth. Ratio of its orbital speed and escape speed will be.

(1) $\frac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) $\sqrt{3}$

(4) $2\sqrt{2}$

Ans. (1)

Sol.

$$\frac{v_0}{v_e} = \frac{\sqrt{\frac{Gm}{r}}}{\sqrt{\frac{2Gm}{r}}} = \frac{1}{\sqrt{2}}$$

Question 21. If I is moment of inertia, F is force, v is velocity, E is energy and L is length then, dimension of $\frac{IFv^2}{EL^4}$ will be:

(1) Energy density

(2) Viscosity

(3) Young modulus

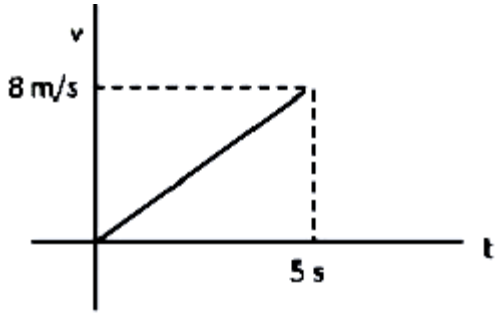
(4) Torque

Ans. (1)

Sol. $\frac{IFv^2}{EL^4} = \frac{(M^1L^2)(M^1L^1T^{-2})(L^1T^{-1})^2}{(M^1L^2T^{-2})(L^4)} = \frac{M^1L^{-2}T^{-2}}{L^3} = M^1L^{-1}T^{-2}$

Energy density

Question 22. Speed time graph of a particle is shown in figure. Find distance travelled by particle in 5 second.



Ans. (20.00)

Sol. Distance = Area of |v| - t graph

$= 1/2 \times 8 \times 5 = 20$ m

Question 23. In displacement method distance of lens and screen is 100 cm Initial Image is obtained on screen. Now lens is displaced 40 cm image formed on screen again. If power of the lens is (100/N) dioptre, then find the value of N:

Ans. (21.00)

Sol. $f = \dfrac{D^2 - d^2}{4D} = \dfrac{100^2 - 40^2}{4(100)} = \dfrac{(100+40)(100-40)}{4(100)}$

$= \dfrac{140 \times 60}{4 \times 100} = \dfrac{14 \times 6}{2 \times 2} = 7 \times 3 = 21$ cm

$P = \dfrac{100}{21} = \dfrac{100}{21} D$

Question 24. Binding energy per nucleon of $^{50}Sn_{120}$ approximately will be. [Atomic mass of Sn_{120} is 120.500 u

and that of 1H is 1.007 u. Mass of neutron = 1.008u, 1u = 931 MeV]

Ans. (3.18 MeV)

Sol. The number of protons is $^{50}Sn_{120}$ = 50 and the number of neutrons = 120 - 50 = 70.

The binding energy of $^{50}Sn_{120}$ is

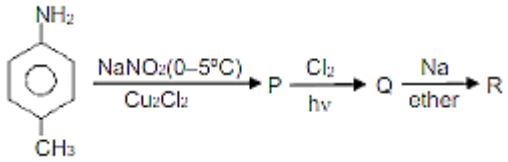
$$= \left[50 \times 1.007\text{u} + 70 \times 1.008\text{u} - 120.500\text{u}\right] c^2 = \left(0.41\text{u}\right)c^2$$

$$= \left(0.41\text{ u}\right)\left(931\text{ MeV / u}\right) = 381.71\text{ MeV}.$$

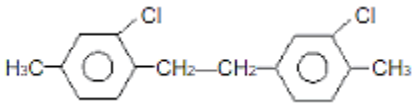
$$\text{Binding energy per nucleon} = \frac{381.71}{120} = 3.18\text{ MeV}$$

Chemistry

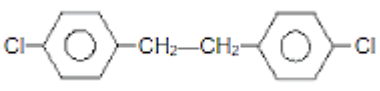
Question 25. Identify end product of following reaction sequence?



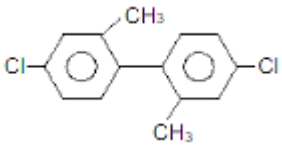
(1)



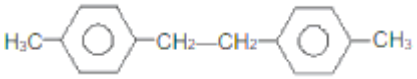
(2)



(3)

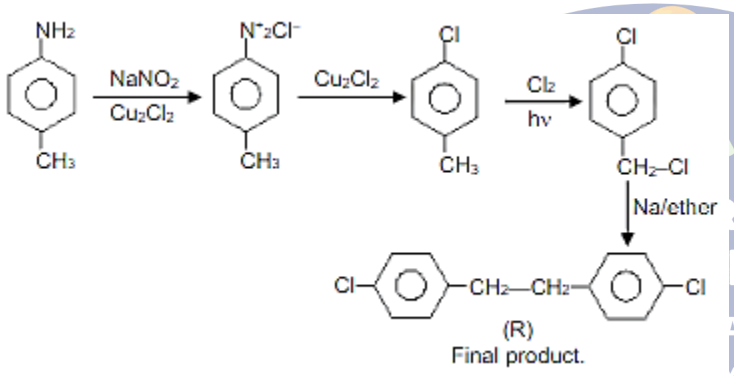


(4)



Answer: (2)

Solution:



Question 26. During roasting and calcination emitted gases produce which of the following effects

(1) Photochemical smog, acid rain

(2) Acid Rain, Global warming

(3) Photochemical smog, Global warming

(4) Acid Rain, ozone depletion

Answer: (2)

Question 27. Synthetic drugs (seldane) Terfenadine work with Histamine.

(1) Increase stimulation of Histamine

(2) Drugs that bind to receptor site and inhibit its natural function

(3) Increase reactivity of Histamine

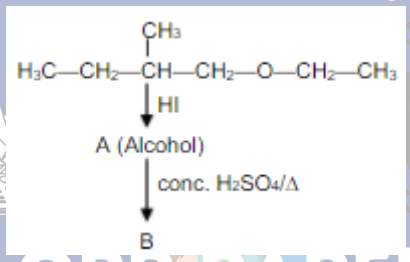
(4) This drugs mimic the natural messenger by switching on the receptor

Answer: (2)

Solution:

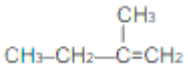
Seldane act as antihistamines and interfere with the natural action of histamine by completing with histamine for binding sites for receptor.

Question 28.

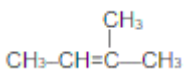


Find out major product of following reaction sequence

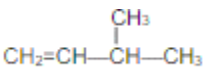
(1)



(2)



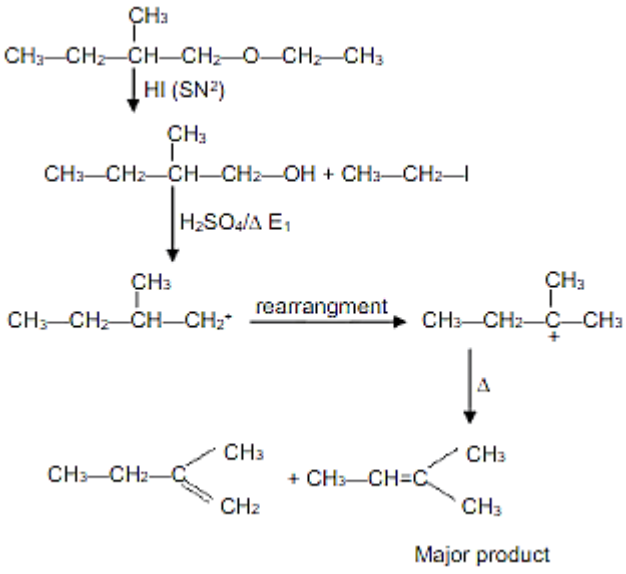
(3)



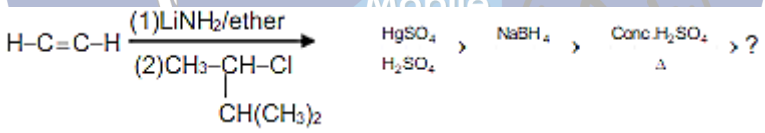
(4) $CH_3 - CH_2 - CH = CH - CH_3$

Answer: (2)

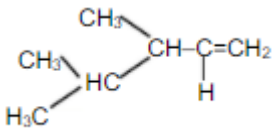
Solution:



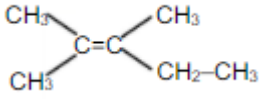
Question 29. Find product of following reaction sequence?



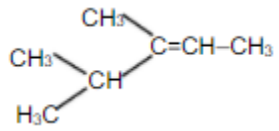
(1)



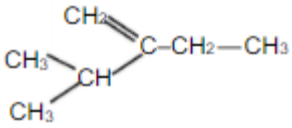
(2)



(3)

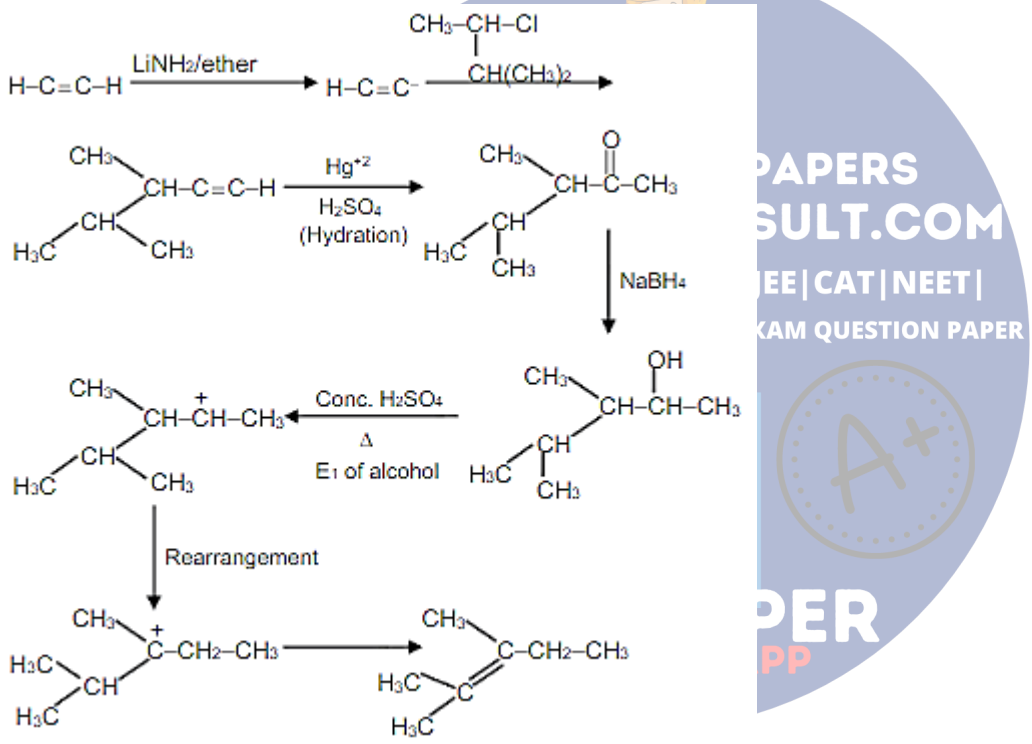


(4)



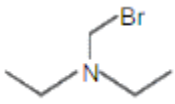
Answer: (2)

Solution:



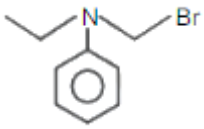
Question 30. Which one is most reactive towards aq. $AgNO_3$

(1)

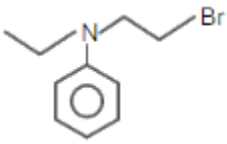


(2)

(3)



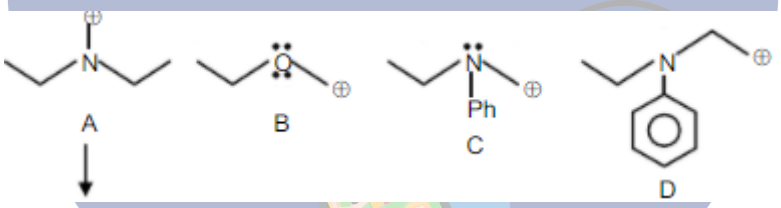
(4)



Answer: (1)

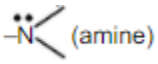
Solution:

Given reaction is examples of SN1 reaction. Which depend upon stability of carbocation.



Most stable by +M

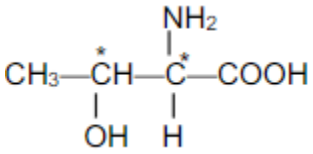
Effect of



Question 31. How many chiral centres are present in Threonine.

Answer: 2.00

Solution:



Threonine have two chiral carbon atom.

Question 32. Which of the following complex show maximum paramagnetism?

[PPh^3 = triphenyl phosphine, OX^{2-} = oxalate, gly- = glycinato]

(1) $[Co(OX)_2(NH_3)_2]^-$

(2) $[Fe(en)(bipy)(NH_3)_2]^{2+}$

(3) $[Pd(gly)(PPh_3)_2]^+$

(4) $[Ti(H_2O)_6]^{3+}$

Answer: (4)

Solution:

	Complex	Electronic configuration	No. of unpaired e$^-$
1	$[Co(OX)_2(NH_3)_2]^-$	$Co^{3+} = 3d_6 \Rightarrow t_2g_6, eg_0$	0
2	$[Fe(en)(bipy)(NH_3)_2]^{2+}$	$Fe^{2+} = 3d_6 \Rightarrow t_2g_6, eg_0$	0
3	$[Pd(gly)(PPh_3)_2]^+$	$Pd^{2+} = 4d_8$	0
4	$[Ti(H_2O)_6]^{3+}$	$Ti^{3+} = 3d_1 \Rightarrow t_2g_1, eg_0$	1

Question 33. Identify the complex in which only one d orbital is used in the hybridization.

(1) $[Ni(CN)_4]^{2-}$

(2) $[Fe(CN)_4]^{3-}$

(3) $[Co(en)_3]^{3+}$

(4) $[FeF_6]^{3-}$

Answer: (1)

Solution:

	Complex	EC	Hybridisation
1	$[Ni(CN)_4]^{2-}$	$Ni^{2+} = 3d_8$	dsp_2
2	$[Fe(CN)_4]^{3-}$	$Fe^{3+} = 3d_5 \Rightarrow t2g2,2,1, eg0,0$	d_2sp_3
3	$[Co(en)_3]^{3+}$	$Co^{3+} = 3d_6 \Rightarrow t2g2,2,2, eg0,0$	d_2sp_3
4	$[FeF_6]^{3-}$	$Fe^{3+} \Rightarrow 3d_5 \Rightarrow t2g1,1,1, eg1,1$	sp_3d_2

Question 34. In hydrogen spectrum shortest wave length for Lyman series line is l, then find longest wave length of Balmer series line in He+ ion spectrum.

(1) l

(2) $\frac{9}{5}l$

(3) $\frac{5}{9}l$

(4) $\frac{4}{9}1$

Answer: (2)

Solution:

For hydrogen atom:

For Lyman series n1 = 1 & n2 = ∞

$$\frac{1}{\lambda_H} = R_H\left[\frac{1}{1} - \frac{1}{\infty}\right] \text{ So, } \lambda = \frac{1}{R_H}$$

For H^{e+} ion

Balmer series n1 = 2 & n2 = 3

$$\frac{1}{\lambda_{He^+}} = R_H \times Z^2\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_{He^+}} = R_H \times 4 \times \frac{5}{36}$$

$$\frac{1}{\lambda_{He^+}} = \frac{5}{9}R_H = \left(\frac{5}{9}\right)\frac{1}{\lambda}$$

$$\left(\lambda_{He^-}\right) = \frac{9}{5}\lambda$$

Question 35. Which of the following process is not endothermic?

(1) $H(g) + e^- \rightarrow H^-(g)$

(2) $Ar(g) + e^- \rightarrow Ar^-(g)$

(3) $O^- + e^- \rightarrow O^{2-}(g)$

(4) $Na(g) \rightarrow Na^+(g) + e^-$

Answer: (1)

Solution:

$H(g) + e^- \xrightarrow{\text{exothermic}} H^-(g)$ $\Delta Heg = -72$ KJ/mol

$O^-(g) + e^- \xrightarrow{\text{endothermic}} O^{2-}(g)$ $\Delta Heg = +744$ KJ/mol

$Ar(g) + e^- \xrightarrow{\text{endothermic}} Ar^-(g)$ $\Delta Heg = +96$ KJ/mol

$Na(g) \xrightarrow{\text{endothermic}} Na+(g) + e^-$ IE = 495.8 KJ/mol

Question 36. Calculate CFSE for complex $[Co(H_2O)3F_3]$ [Given $\Delta 0 < P$]

(1) $-0.4\Delta 0 + 2P$

(2) $-0.4\Delta 0 + P$

(3) $-0.8\Delta 0$

(4) $-0.4\Delta 0$

Answer: (4)

Solution:

$[Co(H_2O)3F_3]$ $Co^{3+} = 3d_6 4s_0 \Rightarrow t2g2,1,1, eg1,1$

$CFSE = [-0.4nt2g + 0.6neg]\Delta 0 + n(P)$

$= [-0.4 \times 4 + 0.6 \times 2]\Delta 0 + 0$

$= -0.4\Delta 0$

Question 37. 5 mole of an ideal gas of volume V is expanded against vacuum to make its volume 2 times, then work done by the gas is

(1) $-RT(V_2 - V_1)$

(2) $-RT\ell n\left(\frac{V_2}{V_1}\right)$

(3) zero

(4) $Cv[T_2 - T_1]$

Answer: (3)

Solution:

$W = -P_{ext}\Delta V$

In expansion against vacuum $P_{ext} = 0$

So work done is zero

Question 38. 100ml solution of each 0.1M AuCl and 0.1M AgCl is electrolysed by passing 1 amp current for 15 min, then which of the following will be deposited?

[Given $Au^+(aqs) + e^- \rightarrow Au$ E0 = 1.69V, $Au^+(aqs) + e^- \rightarrow Au$ $E_0 = 1.69V$]

(1) Only Au

(2) Only Ag

(3) Both Au and Ag

(4) None of Au and Ag

Answer: (1)

Solution:

Change (q)

$$= \frac{it}{96500} F = \frac{1 \times 15 \times 60}{96500} = \frac{900}{96500} = \frac{9}{965} F = 0.0093F$$

No. of moles of $Au^+ = 0.01$ & No. of moles of $Ag^+ = 0.01$

Species with higher value of SRP will get deposited first at cathode.

(1) $Au^+(aqs) + e^- \rightarrow Au(s)$

0.01 0.0093 mole

So only Au will get deposited

Question 39. An alkaline earth metal, sulphate is soluble in water while its hydroxide is not soluble in water and its oxide does not form rock salt structure, then metal is

(1) Be

(2) Mg

(3) Ca

(4) Sr

Answer: (1)

Solution:

$BeSO_4$ Soluble in water

$Be(OH)_2$ Insoluble in water

Structure of BeO is Hexagonal Wurtzite.

Question 39. In which of the following reaction, Hybridisation of underline atom gets changed

(1) $XeF_4 + SbF_5 \rightarrow$

(2) $H_3PO_2 \xrightarrow{\text{disproport ionation}}$

(3) $H_2SO_4 + NaCl \rightarrow$

(4) $NH_3 + BF_3 \rightarrow$

Answer: (1)

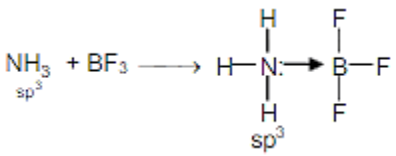
Solution:

(1) $\underset{sp^3d^2}{XeF_4} + SbF_5 \rightarrow \left[\underset{sp^3d}{XeF_3}\right]^+ \left[SbF_6\right]^-$

(2) $\underset{sp^3}{H_3PO_2} \rightarrow \underset{sp^3}{H_3PO_4} + PH_3$

(3) $\underset{sp^3}{H_3PO_2} + 2NaCl \rightarrow \underset{sp^3}{Na_2SO_4} + 2HCl$

(4)



Question 40. Given

(1) $A \rightleftharpoons B + C$ Keq (1)

(1) $B + C \rightleftharpoons P$ Keq (2)

then Keq for reaction $A \rightleftharpoons P$ is

(1) Keq (1). Keq (2)

(2) $\frac{K_{eq}(1)}{K_{eq}(2)}$

(3) Keq (1) + Keq (2)

(4) Keq (1) - Keq (2)

Answer: (1)

Solution:

On adding Reaction 1st and Reaction 2nd we get.

$A \rightleftharpoons P$ Keq = Keq (1). Keq (2)

Question 41. In colloidal solution of blue ink following reagent are mixed H_2O, Egg, CH_3COOH and HCl then which of the above reagent ensure the stability of Blue ink

(1) H_2O

(2) Egg albumen

(3) CH_3COOH

(4) HCl

Answer: (2)

Solution:

Blue ink is a colloidal sol, so it can be stabilised by material like natural gum or Egg albumen.

Question 42. Osmotic pressure of NaCl solution is 0.1 atm and Glucose solution is 0.2 atm. If 1 L of NaCl solution and 2 L of Glucose solution is mixed at same temperature, then osmotic pressure of resulting solution is 'X' $\times 10^{-3}$ atm. Then value of 'X' in nearest integer is

Answer: 167

Solution:

$$\Pi_{final} = i\ CRT = i\left[\frac{n}{V}\right]RT$$

$$\Pi_{final} = \frac{(\pi_1 V_1)+(\pi_2 V_2)}{V_1 + V_2}$$

$$\Pi_{final} = \frac{(0.1\times 1)+(0.2\times 2)}{3}$$

$$= \frac{(0.1+0.4)}{3} = \frac{0.5}{3} = \frac{500}{3}\times 10^{3}\ \text{atm}$$

So X = 167

Question 43. If temperature changes from 27°C to 42°C then no. of molecule having energy greater than threshold energy

become five times, then find activation energy (Ea) of reaction (in kJ) [Given in 5 = 1.6094 & R = 8.314 $\frac{J}{Mole \times k}$]

Answer: 84.30kJ

Solution:

$$k - Ae^{-} \frac{Ea}{RT}$$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln(5) = \frac{Ea}{8.314}\left[\frac{1}{300} - \frac{1}{315}\right]$$

$$1.6094 = \frac{Ea}{8.314}\left[\frac{15}{300 \times 315}\right]$$

$$Ea = 84297.55$$

$$= 84.2975 \text{ kJ}$$

$$= 84.30 \text{ kJ}$$

Question 44. In 100 mL, 0.1 N Na_2CO_3 x H_2O solution. Mass of solute is 1.43 gram, then value of X is

Answer: 10.00

Solution:

Equivalent of solute = 0.1 × 0.1

Mole of solute ($Na2CO_3$ x H_2O) = $[0.1 \times 0.1]\frac{1}{2}$

Mass of Na_2CO_3 x H_2O = $[0.1 \times 0.1]\frac{1}{2} \times [106 + 18x] = 1.43$

$\Rightarrow [106 + 18x] = 286$

$18x = 180$

$x = 10$

Question 45. For the following redox reactions

(1) $2Fe^{2+} + H_2O_2 \rightarrow xA + yB$

(2) $2MnO_4 + 6H^+ + 5H_2O_2 \rightarrow x'A' + y'B' + z'C'$

Find the sum of coefficient $(x + y + x' + y' + z')$

Answer: 19.00

Solution:

(1) $2Fe^{2+} + H_2O_2 + 2H^+ \rightarrow 2Fe^{3+} + 2H_2O$

(2) $2MnO_4 + 5H_2O_2 + 6H^+ \rightarrow 2Mn^{2+} + 5O_2 + 8H_2O$

Find the sum of coefficient $(x + y + x' + y' + z') = 2 + 2 + 2 + 5 + 8 = 19$

Mathematics

Question 46. If a1, a2, a3, ……. an are in Arithmetic Progression, whose common difference is an integer such that a1 = 1, an = 300 and n $n \in [15, 50]$, then (Sn-4, an-4) is

(1) (2491, 247)

(2) (2490, 248)

(3) (2590, 249)

(4) (248, 2490)

Ans. (2)

Sol. an = a1 + (n -1)d $\Rightarrow 300 = 1 + (n-1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{13\times 23}{(n-1)} = \text{integer}$$

So $n-1 = \pm 1, \pm 13, \pm 23, \pm 299$

$\Rightarrow$ n = 14, -12, 24, -22, 300, -298, 2, 0

But $n \in [15, 20] \Rightarrow n = 24 \Rightarrow d = 13$

Hence

$$S_{n-4} = S_{20} = \frac{20}{2}\left[2(1)+(20-1)(13)\right] = 10[2+247] = 2490$$

an-4 = a20 = a1 + 19d

= 1 + 19 × 13

= 1+ 247

= 248

Question 47. If $\lim_{t\to x} \frac{x^2 f^2(t) - t^2 f^2(x)}{t-x} = 0$ and f(1) = e then solution of f(x) = 1 is

(1) $\frac{1}{e}$

(2) $\frac{1}{2e}$

(3) e

(4) 2e

Ans. (1)

Sol. $\lim_{t\to x}\frac{x^2f^2(t)-t^2f^2(x)}{t-x}=0$

Using L'Hospital

$\lim_{t\to x}\frac{x^2.2f(t).f'(t)-2tf^2(x)}{1}=0$

$x^2\ 2f(x)\ f'(x)-2x\ f^2(x)=0$

$2x\ f(x)\ [xf'(x)-f(x)]=0$

$f(x)\neq 0$

so $xf'(x)=f(x)$

$x\frac{dy}{dx}=y$

$\frac{1}{y}dy=\frac{1}{x}dx$

Integration $\ln y=\ln x+\ln c$

$y=cx \Rightarrow f(x)=cx$

Now f(1) = c = e

So f(x) = ex

Now f(x) = 1

$ex = 1 \Rightarrow x = \frac{1}{e}.$

Question 48. Minimum value of 2sinx + 2cosx is

(1) $2^{1-\frac{1}{\sqrt{2}}}$

(2) $2^{1+\frac{1}{\sqrt{2}}}$

(3) $2^{1+\sqrt{2}}$

(4) $2^{1-\sqrt{2}}$

Ans. (1)

Sol. Using A.M. ≥ G.M.

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2.^{\sin x}.2^{\cos x}}$$

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq 2^{\frac{\sin x + \cos x}{2}} \qquad \text{(i)}$$

Now $-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$

So $-\frac{1}{\sqrt{2}} \leq \frac{\sin x + \cos x}{2} \leq \frac{1}{\sqrt{2}}$

Minimum value of $2^{\frac{\sin x + \cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$

So by (i)

So, $\frac{2^{\sin x}+2^{\cos x}}{2}=2^{-\frac{1}{\sqrt{2}}}$

Hence, $2^{\sin x}+2^{\cos x}\geq 2^{1}.2^{-\frac{1}{\sqrt{2}}}\geq 2^{1-\frac{1}{\sqrt{2}}}$

Question 49. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ then the value of $\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2+\left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2+\left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$ is

Ans. (18.00)

Sol. Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

$\hat{i}\times(\vec{a}\times\hat{i})=(\hat{i}.\hat{i})\vec{a}-(\vec{a}.\hat{i})\hat{i}=y\hat{j}+z\hat{k}$

Similarly $\hat{j}\times(\vec{a}\times\hat{j})=x\hat{i}+z\hat{k}$ and $\hat{k}\times(\vec{a}\times\hat{k})=x\hat{i}+y\hat{k}$

$\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2+\left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2+\left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$

$=\left|y\hat{j}+z\hat{k}\right|^2+\left|x\hat{i}+z\hat{k}\right|^2+\left|x\hat{i}+y\hat{j}\right|^2=2\left|a\right|^2=2(9)=18$

Question 50. $\int_0^n\{x\}dx,\int_0^n[x]dx$ and $10\left(n^2-n\right)$ are in Geometric progression, where $\{x\}$ & $[x]$ represents fractional part function and greatest integral function respectively, find n if
$n\in N$ and $n>1$.

Ans. (21.00)

Sol. $\int_0^n \{x\}\,dx = n\int_0^1 x\,dx = n\left(\frac{x^2}{2}\right)_0^1 = \frac{n}{2}$

And $\int_0^n [x]dx = \int_0^n \left(x - \{x\}\right)dx = \left(\frac{x^2}{2}\right)_0^n - \int_0^n \{x\}\,dx = \frac{n^2}{2} - \frac{n}{2}$

Now, $\frac{n}{2}, \frac{n^2-n}{2}$ and $10(n^2-n)$ are in Geometric Progression

$$\Rightarrow \frac{n^2(n-1)^2}{4} = 5.n^2(n-1) \Rightarrow n-1 = 20 \Rightarrow n = 21$$

Question 51. The ratio of three consecutive terms in expansion of $(1 + x)^{n+5}$ is 5 : 10 : 14, then greatest coefficient is

(1) 252

(2) 462

(3) 792

(4) 320

Ans. (2)

Sol. Let three consecutive term are T_r, T_{r+1}, T_{r+2}

Hence $\frac{T_r}{T_{r+1}} = \frac{5}{10}$ and $\frac{T_{r+1}}{T_{r+2}} = \frac{10}{14}$

$\frac{T_{r+1}}{T_r} = 2$ $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{7}$

$$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = 2 \qquad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{7}{5}$$

$$\frac{(n+5)-r+1}{r} = 2 \qquad \frac{(n+5)-(r+1)+1}{r+1} = \frac{7}{5}$$

$$n - r + 6 = 2r \qquad \frac{n-r+5}{r+1} = \frac{7}{5}$$

$$n - 3r + 6 = 0 \qquad \text{.....(i)}$$

$$5n - 5r + 25 = 7r + 7$$

$$5n - 12r + 18 = 0 \qquad \text{...(ii)}$$

Multiply equation (i) by 5

$$5n - 15r + 30 = 0$$

$$5n - 12r + 18 = 0$$

$$- \quad + \quad -$$

$$-3r + 12 = 0 \Rightarrow (r = 4, n = 6)$$

Hence greatest coefficient will be of middle term = $n+{}^5C_5$ $={}^{11}C_5 = 462$

Question 52. There are 6 multiple choice questions in a paper each having 4 options of which only one is correct. In how many ways a person can solve exactly four correct, if he attempted all 6 questions.

(1) 134

(2) 135

(3) 136

(4) 137

Ans. (2)

Sol. No. of ways of giving wrong answer = 3

Required no. of ways = ${}^{6}C_{4}$ (1)4 × (3)2

= 15(9) = 135

Question 53.

class	0-10	10-20	20-30
f	2	x	2

If variance of variable is 50 than x =

(1) 5

(2) 6

(3) 4

(4) 3

Ans. (3)

Sol.

xi	5	15	25
fi	2	x	2

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{10 + 15x + 50}{4 + x}$$

$$= \frac{60 + 15x}{4 + x} = 15$$

$$\sigma^2 = 50 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$$

$$50 = \frac{50 + 225x + 1250}{4 + x} - (15)^2$$

$$50 = \frac{1300 + 225x}{4 + x} - 225$$

$$\Rightarrow 275\ (4 + x) = 1300 + 225x \Rightarrow 50x = 200 \Rightarrow x = 4$$

Question 54. Two persons A and B play a game of throwing a pair of dice until one of them wins. A will win if sum of numbers on dice appear to be 6 and B will win, if sum is 7. What is the probability that A wins the game if A starts the game?

(1) $\frac{31}{61}$

(2) $\frac{30}{61}$

(3) $\frac{29}{61}$

(4) $\frac{32}{61}$

Ans. (2)

Sol. sum 6 $\rightarrow$ (1, 5), (5, 1), (3, 3), (2, 4), (4, 2)

Sum 7 $\rightarrow$ (1, 6), (6, 1), (5, 2), (2, 5), (3, 4), (4, 3)

$$P(A \text{ wins})$$
$$= P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A})P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A) + ...$$

This is infinite G.P. with common ratio $P(\bar{A}) \times P(\bar{B})$

$$\text{Probability of A wins} = \frac{P(A)}{1 - P(\bar{A})P(\bar{B})}$$

$$= \frac{\frac{5}{36}}{1 - \frac{31}{36}.\frac{30}{36}} = \frac{30}{61}$$

Question 55. If ω is an imaginary cube roots of unity such that $(2+\omega)^2 = a + b\omega, a, b \in R$ then value of a + b is

(1) 7

(2) 6

(3) 8

(4) 5

Ans. (2)

Sol. $(2+\omega)^2 = a + b\omega$

$4 + \omega^2 + 4\omega = a + b_{\omega}$ $\because$ $1 + \omega^2 = -\omega$

$3 + 3\omega = a + b\omega$

$$(a-3)+\omega(b-3)=0$$

$$(a-3)+\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)(b-3)=0$$

$$(a-3)-\frac{1}{2}(b-3)+i\frac{\sqrt{3}}{2}(b-3)=0$$

compare real and imaginary part from both sides

$$(a-3)-\frac{1}{2}(b-3)=0 \text{ and } b-3=0 \Rightarrow b=3 \text{ and } a=3$$

Hence a + b = 6

Question 56. Centre of a circle S passing through the intersection points of circles $x^2 + y^2 - 6x = 0$ & $x^2 + y^2 - 4y = 0$ lies on the line $2x - 3y + 12 = 0$ then circle S passes through

(1) (-3, 1)

(2) (-4, -2)

(3) (1, 2)

(4) (-3, 6)

Ans. (4)

Sol. By family of circle, passing through intersection of given circle will be member of

$$S_1 + \lambda S_2 = 0 \text{ family } (\lambda \neq 1)$$

$(x^2 + y^2 - 6x) + \lambda\left(x^2 + y^2 - 4y\right) = 0$

$(\lambda+1)x^2 + (\lambda+1)y^2 - 6x - 4\lambda y = 0$

$x^2 + y^2 - \frac{6}{\lambda+1} \times -\frac{4\lambda}{\lambda+1} y = 0$

Centre $\left(\frac{3}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right)$

Centre lies on $2x - 3y + 12 = 0$

$2\left(\frac{3}{\lambda+1}\right) - 3\left(\frac{2\lambda}{\lambda+1}\right) + 12 = 0$

$6\lambda + 18 = 0$

$\lambda = -3$

Circle $x^2 + y^2 + 3x - 6y = 0$

Question 57.

$$\int_{\pi/6}^{\pi/3} \tan^3 x.\sin^2 3x\left(2\sec^2 x.\sin^2 3x + 3\tan x.\sin 6x\right)dx$$

(1) $-\frac{1}{36}$

(2) $-\frac{1}{72}$

(3) $-\frac{1}{18}$

(4) $\frac{1}{36}$

Ans. (3)

Sol.

$$\int_{\pi/6}^{\pi/3}\left(\frac{\frac{d}{dx}\left(\tan^4 x\right)}{2}.\sin^4 3x+\tan^4 x.\frac{\frac{d}{dx}\left(\sin^4 3x\right)}{2}\right)$$

$$=\frac{1}{2}\int_{\pi/6}^{\pi/3}\frac{d}{dx}\left(\tan^4 x.\sin^4 3x\right)dx$$

$$=\frac{1}{2}\left[\tan^4 x.\sin^4 3x\right]_{\pi/6}^{\pi/3}=\frac{1}{2}\left[(3)^4\times 0-\frac{1}{\left(\sqrt{3}\right)^4}\right]=-\frac{1}{2}\times\frac{1}{9}=-\frac{1}{18}.$$

Question 58. From a pt 200 m above a lake, the angle of elevation of a cloud is 30º and the angle of depression of its reflection in take is 60º then the distance of cloud from the point is

(1) 400 m

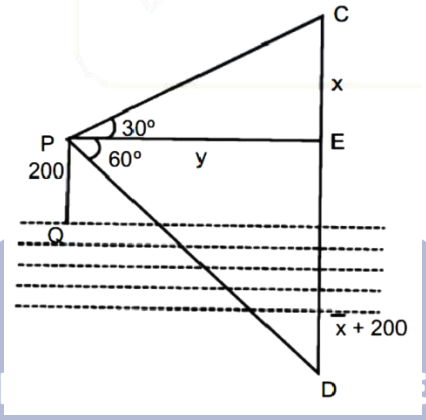
(2) $400\sqrt{2}$m

(3) $400\sqrt{3}$m

(4) 200 m

Ans. (1)

Sol.



$$\tan 30^\circ = \frac{x}{y} = \frac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}x \quad \ldots\ldots(i)$$

$$\text{and } \tan 60^\circ = \frac{x + 400}{y} \Rightarrow \sqrt{3}y = x + 400 \quad \ldots\ldots(ii)$$

Solving (i) and (ii), we get

$2x = 400, x = 200$

$$\sin 30^\circ = \frac{x}{PC} = \frac{200}{PC} \Rightarrow PC = 400$$

Question 59. The contrapositive of statement:

"If f(x) is continuous at x = a then f(x) is differentiable at x = a

(1) It f(x) is continuous at x = a then f(x) is not continuous at x = a

(2) If f(x) is not differentiable at x = a then f(x) is not continuous at x = a

(3) If f(x) is differentiable at x = a then f(x) is continuous at x = a

(4) If f(x) is differentiable at x = a then f(x) is not continuous

Ans. (2)

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$.

Question 60. If equation of directrix of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x=4$, then normal to the ellipse at point $(1,\beta),(\beta>0)$ passes through the point (where eccentricity of the ellipse is $\frac{1}{2}$)

(1) $\left(1,\frac{3}{2}\right)$

(2) $\left(-1,\frac{3}{2}\right)$

(3) (-1, -3)

(4) (3, -1)

Ans. (1)

Sol. $\frac{a}{e} = 4 \Rightarrow a = 4e \Rightarrow a = 2$

$b2 = a2\,(1 - e2) = 3$

$(1,\beta)$ lies on $\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow \frac{1}{4} + \frac{\beta^2}{3} = 1 \Rightarrow \beta^2 = \frac{9}{4} = \frac{3}{2} (\because \beta > 0)$

Normal at $(1,\beta) \Rightarrow \frac{a^2 x}{1} - \frac{b^2 y}{\beta} = a^2 - b^2 \Rightarrow 4x - \frac{3y}{3/2} = 1$

So equation of normal is $4x - 2y = 1$

Question 61. If points A and B lie on x-axis and points C and D lie on the curve $y = x^2 - 1$ below the x-axis then maximum area of rectangle ABCD is

(1) $\frac{4\sqrt{3}}{3}$

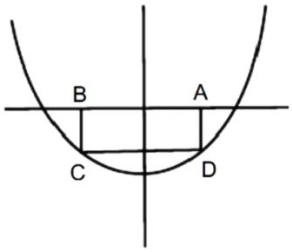
(2) $\frac{4\sqrt{3}}{9}$

(3) $\frac{4\sqrt{3}}{27}$

(4) $\frac{8\sqrt{3}}{9}$

Ans. (2)

Sol.



$A(\alpha,0), B(-\alpha,0)$

$\Rightarrow D(\alpha,\alpha^2-1)$

Area (ABCD) = (AB)(AD)

$\Rightarrow S=(2\alpha)(1-\alpha^2)=2\alpha-2\alpha^3$

$\frac{ds}{d\alpha}=2-6\alpha^2=0\Rightarrow a^2=\frac{1}{3}\Rightarrow\alpha=\frac{1}{\sqrt{3}}$

Area $2\alpha-2\alpha^3=\frac{2}{\sqrt{3}}-\frac{2}{3\sqrt{3}}=\frac{4}{3\sqrt{3}}$

Question 62. If α,β are roots of $x^2-x+2\lambda=0$ and α,γ are roots of $3x^2-10x+27\lambda=0$ then value of $\frac{\beta\gamma}{\lambda}$ is

(1) 27

(2) 18

(3) 9

(4) 15

Ans. (2)

Sol. Given $3\alpha^2-10\alpha+27\lambda=0$ ..(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ ..(ii)

Subtract $-7\alpha + 21\lambda = 0$

$3\lambda = \alpha$

By (ii) $9\lambda^2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 0, \frac{1}{9}$

$\therefore$ given equation are $x^2 - x + \frac{2}{9} = 0$ and $3x^2 - 10x + 3 = 0$

$\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \alpha = \frac{1}{3}, \gamma = 3$

$\therefore \frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}.3}{\frac{1}{9}} = 18$

Question 63. PQ is a diameter of circle $x^2 + y^2 = 4$. If perpendicular distances of P and Q from line $x + y = 2$ are α and β respectively then maximum value of $\alpha\beta$ is

Ans. (2)

Sol. Let $P(2\cos\theta, 2\sin\theta) \therefore Q(-2\cos\theta, -2\sin\theta)$

Given line $x + y - 2 = 0$

$\therefore \alpha = \frac{|2\cos\theta + 2\sin\theta - 2|}{\sqrt{2}}$

$$\beta = \frac{\left|-2\cos\theta - 2\sin\theta - 2\right|}{\sqrt{2}}$$

$$\therefore \alpha\beta = \sqrt{2}\left(\cos\theta + \sin\theta - 1\right).\sqrt{2}\left(\cos\theta + \sin\theta + 1\right)$$

$$= 2\left|\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1\right| = 2\left|\sin 2\theta\right|$$

Max $\left|\sin 2\theta\right| = 1$

$\therefore$ maximum $\alpha\beta = 2.$

Question 64. If $\frac{dy}{dx} - \frac{y-3x}{\ln(y-3x)} = 3$, then

(1) $\frac{\ln(y-3x)}{2} = x + c$

(2) $\frac{\ln^2(y-3x)}{2} = x + c$

(3) $\frac{\ln(y-3x)}{2} = x^2 + c$

(4) $\frac{\ln^2(y-3x)}{2} = x^2 + c$

Ans. (2)

Sol. $\frac{dy}{dx} - \frac{y-3x}{\ln(y-3x)} - 3 = 0$

$$\frac{dy}{dx}-3=\frac{y-3x}{\ln(y-3x)}$$

$$\frac{d}{dx}(y-3x)=\frac{y-3x}{\ln(y-3x)}$$

$$\int\frac{\ln(y-3x)}{(y-3x)}d(y-3x)=\int dx$$

Let $\ln(y-3x)=t$

$$\frac{1}{(y-3x)}d(y-3x)=dt$$

$$\int tdt=\int dx$$

$$\frac{t^2}{2}=x+c$$

$$\frac{(\ln(y-3x))^2}{2}=x+c$$

Question 65. The distance of point (1, -2, -3) from plane x – y + z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

(1) 7

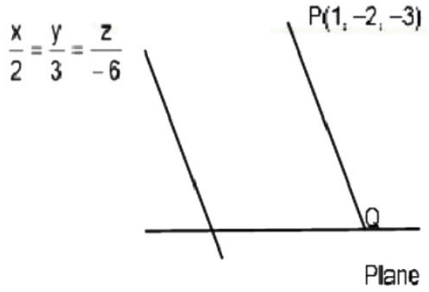
(2) $\frac{1}{7}$

(3) 1

(4) 5

Ans. (4)

Sol.



Equation PQ

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z+3}{-6}=\lambda$$

Let $Q \equiv (2\lambda+1, 3\lambda-2, -6\lambda-3)$

Q lies on x - y + z = 5

$$\equiv (2\lambda+1)-(3\lambda-2)+(-6\lambda-3)=5 \quad \Rightarrow \lambda=-\frac{5}{7}$$

$$\Rightarrow Q \equiv \left(-\frac{3}{7}, -\frac{29}{7}, \frac{9}{7}\right)$$

$$PQ=\sqrt{\left(1+\frac{3}{7}\right)^2+\left(-2+\frac{29}{7}\right)^2+\left(-3-\frac{9}{7}\right)^2}$$

$$=\sqrt{\frac{100}{49}+\frac{225}{49}+\frac{900}{49}}=\sqrt{\frac{1225}{49}}=\frac{35}{7}=5$$

Question 66. If $f(x)=\begin{cases}\frac{1}{2}(|x|-1), & (|x|>1)\\ \tan^{-1}x, & |x|\le 1\end{cases}$ then f(x) is

(1) continuous for $x\in R-\{0\}$

(2) continuous for $x\in R-\{0,1,-1\}$

(3) not continuous for $x\in\{-1,0,1\}$

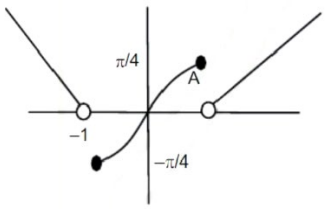
(4) f(x) is continuous for $x\in R-\{1,-1\}$

Ans. (4)

Sol.

$$f(x)=\begin{cases}\frac{|x|-1}{2}, & |x|>1\\ \tan^{-1}x, & |x|\le 1\end{cases}$$

Graph of f(x) is



f(x) is not continuous at x = -1, 1

Question 67. Suppose X1, X2,……..X50 are 50 sets each having 10 elements and Y1, Y2, …….Yn are n sets each having 5 elements. $\bigcup_{i=1}^{50}X_i=\bigcup_{i=1}^{n}Y_i=Z$ and each element of Z

belong to exactly 25 of Xi and exactly 6 of Yi then value of n is

(1) 20

(2) 22

(3) 24

(4) 26

Ans. (3)

Sol. $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = Z \quad \therefore \frac{10\times 50}{25} = \frac{5n}{6} \quad \Rightarrow n = 24$

Question 68. Let A is 3×3 matrix such that Ax1 = B1, Ax2 = B2, Ax3 = B3 where

$$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix} \quad x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix} \quad x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$B_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad B_2 = \begin{bmatrix}0\\2\\0\end{bmatrix} \quad B_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$$

Then find |A|

(1) 0

(2) 1

(3) 2

(4) 3

Ans. (3)

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$Ax_1 = B_1$

$a_1 + a_2 + a_3 = 1$

$b_1 + b_2 + b_3 = 0$

$c_1 + c_2 + c_3 = 0$

Similar $2a_2 + a_3 = 0$ and $a_3 = 0$

$2b_2 + b_3 = 2$ $\quad b_3 = 0$

$2c_2 + c_3 = 0$ $\quad c_3 = 2$

$\therefore a_2 = 0, b_2 = 1, c_2 = -1,$

$a_1 = 1, b_1 = -1, c_1 = -1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \therefore |A| = 2$$