JEE MAIN 2020 6th SEPT SHIFT 2

Physics

Question 1. Find distance of centre of mass of solid hemisphere of radius 8 cm from centre

(1) 3 cm

(2) $\frac{3}{8}$cm

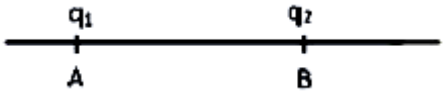
(3) 4 cm

(4) 2 cm

Ans. (1)

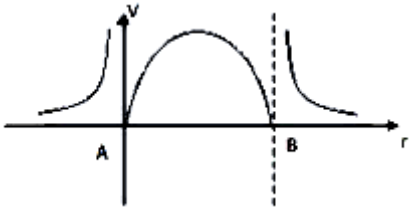
Sol. $h_{cm} = \frac{3R}{8} = \frac{3\times 8}{8} = 3\text{cm}$

Question 2. Two charge +q1 and +q2 placed on x-axis at position A and B as shown in figure.

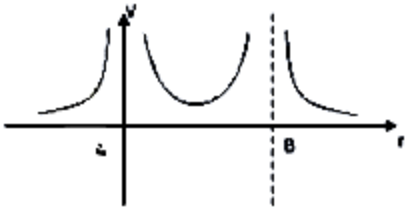


Which graph best represent the potential as a function of distance r from charge q1

(1)



(2)



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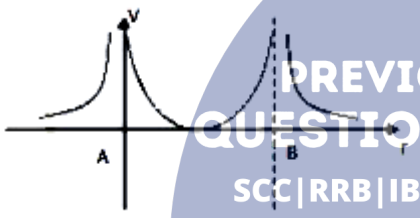
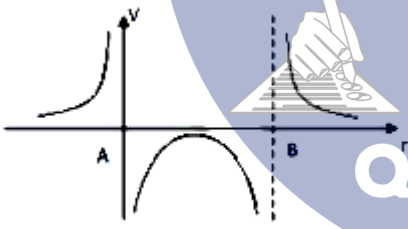


(4)



Ans. (2)

Sol. $V = \frac{kq_1}{r} + \frac{kq_2}{x}$

r = distance from charge q1

x = distance from charge q2

Question 3. Magnetic field of an electromagnetic wave is $\vec{B} = 12\times10^{-9}\sin(kx-\omega t)\hat{k}(T)$. The equation of corresponding electric field should be:

(1) $\vec{E} = 36\sin(kx-\omega t)\hat{j}\ N/C$

(2) $\vec{E} = 3.6\sin(kx-\omega t)\hat{j}\ N/C$

(3) $\vec{E} = 3.6\sin(kx-\omega t)(-\hat{j})\ N/C$

(4) $\vec{E} = 36\sin(kx-\omega t)(-\hat{j})\ N/C$

Ans. (2)

Sol. $E_0 = B_0 \times C = 12\times10^{-9}\times3\times10^{8} = 3.6$

As the light is propagating in x direction

& $(\hat{E}\times\hat{B}) \parallel \hat{C}$

$\therefore \vec{E}$ should be in $\hat{j}$ direction

So electric field $\vec{E} = E_0\sin(kx-\omega t)\hat{j}$

$\vec{E} = 3.6\sin(kx-\omega t)\hat{j}$

Question 4. Rain is falling vertically when car is at rest. When car moves with speed v, rain appears at 60° with horizontal when car moves with speed $(\beta+1)v$, rain appears at 45° with the horizontal. Find value of β:

(1) 3

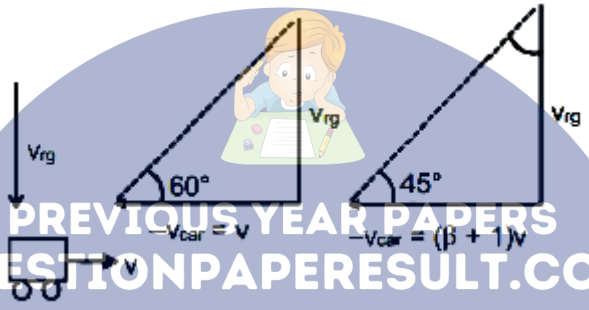
(2) 0.732

(3) 2

(4) 0.5

Ans. (2)

Sol.



$$\tan 60° = \frac{v_{rg}}{v} \quad \ldots. (1)$$

$$\tan 45° = \frac{v_{rg}}{(\beta+1)v} \quad \ldots.. (2)$$

$$\sqrt{3}v = (\beta+1)v$$

$$\beta = \sqrt{3} - 1 = 0.732$$

Question 5. An equi-concave lens has power P. Find power of plano-concave lens, when given lens is cut in such a way that two plano-concave lens are formed:

(1) P

(2) P/2

(3) 2P

(4) P/4

Ans. (2)

Sol. $1/f = (n-1)[(-1/R) - (1/R)] = -(n-1)(2/R)$

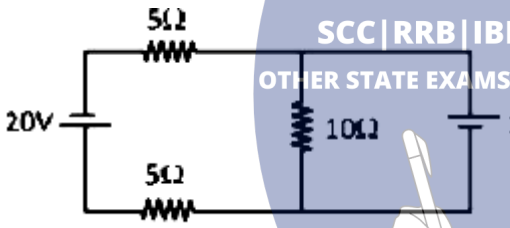
$P = -R/2(n-1)$

$1/f' = (n-1)[(-1/R)-(0)=-(n-1)/R$

$P'=-R/(n-1)$

$P'=P/2$

Question 6. Find current in 10V cell.



(1) 2A

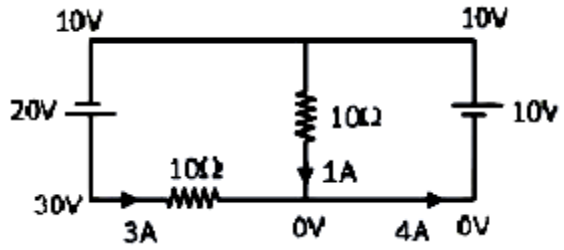
(2) 3A

(3) 4A

(4) 8A

Ans. (3)

Sol.



So current in 10V cell is 4A

Question 7. In a electric circuit ammeter and voltmeter are used to verify ohm's law which of the following is true.

(1) Ammeter connected in series and voltmeter connected in parallel

(2) Voltmeter connected in series and ammeter connected in parallel

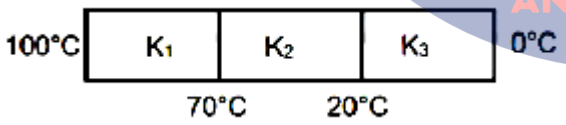
(3) Both can be connected in parallel

(4) Both can be connected in series

Ans. (1)

Sol. Theory Based

Question 8. Steady state temperature are shown in the diagram. Find ratio of thermal conductivity K1/K2 if length of all rods are same.



(1) 3 : 2

(2) 5 : 3

(3) 4 : 2

(4) 5 : 2

Ans. (2)

Sol. $K_1(100-70)=K_2(70-20)=K_3(20-0)$

$K_1 30=K_2(50)=K_3(20)$

$\Rightarrow K_1:K_2:K_3=10:6:15$

Question 9. A particle of mass m moving with speed v collides elastically with another particle of mass 2m. Find speed of smaller mass after head on collision:

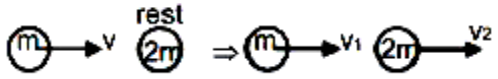
(1) $\frac{v}{3}$

(2) $-\frac{v}{3}$

(3) $\frac{2v}{3}$

(4) $-\frac{2v}{3}$

Ans. (2)

Sol. 

Apply conservation of linear momentum

mv = mv1 + 2mv2

$\frac{v_2-v_1}{v}=1$

Alternate:

$$v_1 = \frac{2m_2u_2}{m_1+m_2} + \frac{m_1-m_2}{m_1+m_2}u_1$$

$$= \frac{2\times(2m)(0)}{3m} + \frac{(m-2m)v}{3m} = -\frac{v}{3}$$

Question 10. Half-life of C19 is 5700 years. Find its decay constant.

(1) 12.16 × 10-5 year-1

(2) 12.16 × 10-3 year-1

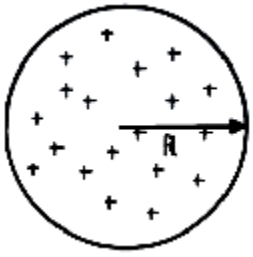
(3) 8.52 × 10-5 year-1

(4) 8.53 × 10-3 year-1

Ans. (1)

Sol. $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6932}{5700} = 12.16\times10^{-5}\ \text{year}^{-1}$

Question 11. Charge density of a sphere of radius R is $\rho = \frac{\rho_0}{r}$, where r is distance from centre of sphere. Total charge of sphere will be



(1) $\rho_0\pi R^2$

(2) $2\rho_0 \pi R^2$

(3) $3\rho_0 \pi R^2$

(4) $4\rho_0 \pi R^2$

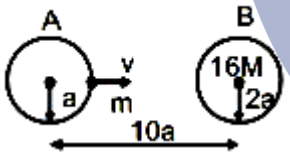
Ans. (2)

Sol. $Q = \int \rho dv = \int \rho 4\pi r^2 dr$

$$= \int \frac{\rho_0}{r} 4\pi r^2 dr = \int_0^R \rho_0 4\pi r dr$$

$$Q = 2\rho_0 \pi R^2$$

Question 12.



Two planets of mass M and 16M of radius a and 2a respectively, are at distance 10a. Find minimum speed of a particle of mass m at surface of smaller planet so that it can reached from smaller planet to larger planet.

(1) $\sqrt{\frac{5}{9}\frac{GM}{a}}$

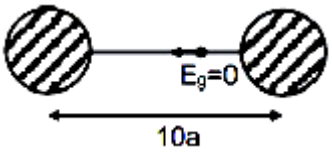
(2) $\sqrt{\frac{4}{9}\frac{GM}{a}}$

(3) $\sqrt{\frac{3}{9}\frac{GM}{a}}$

(4) $\sqrt{\frac{2}{9}\frac{GM}{a}}$

Ans. (1)

Sol.



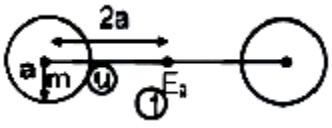
$$\frac{GM}{r^2} = \frac{16\,GM}{(10a-r)^2}$$

$$\frac{1}{r} = \frac{4}{10a-r}$$

$$10a - r = 4r$$

$$5r = 10a$$

$$r = 2a$$



$$U_i + K_i = K_{min} + U_f$$

$$-\frac{GMm}{a} - \frac{G16Mm}{9a} + \frac{1}{2}mv^2 = 0 - \frac{GMm}{2a} - \frac{G16Mm}{8a}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{a} - \frac{GMm}{2a} + \frac{G16Mm}{9a} - \frac{G16Mm}{8a}$$

$$= \frac{GMm}{2a} + \frac{16GMm}{a}\left(\frac{1}{9} - \frac{1}{8}\right) = \frac{GMm}{2a} - \frac{16\,GMm}{9\times 8a} = \frac{GMm}{2a} - \frac{2GMm}{9a}$$

$$= \frac{GMm}{a}\left[\frac{1}{2} - \frac{2}{9}\right];$$

$$\frac{1}{2}mv^2 = \frac{5}{18}\frac{GMm}{a}$$

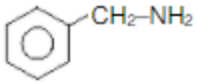
$$v = \sqrt{\frac{5}{9}\frac{GM}{a}}$$

Chemistry

Question 13. Which of the following major product can be obtained from Gabriel phthalimide synthesis?

(1) $CH_2 = CH - NH_2$

(2)



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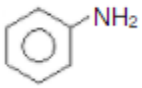

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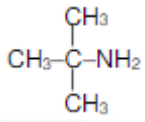
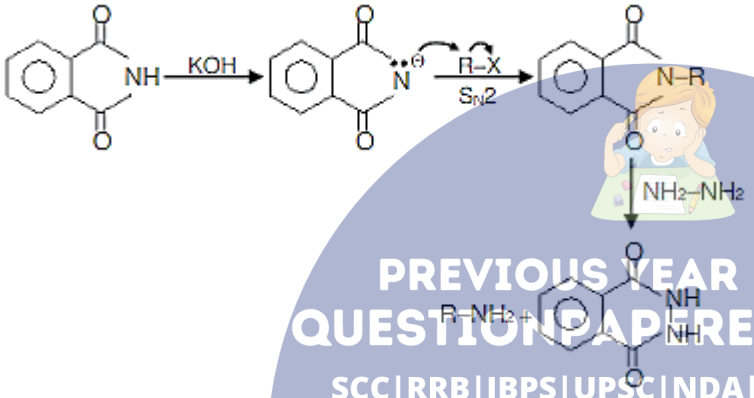


Answer: (2)

Solution:

From Gabriel phthalimide reaction, 1o Amine can be prepared.



Question 14.

Column – 1 (Polymers)	Column – 2 (Monomers)
(I) Natural rubber	(a) Styrene
(II) Neoprene	(b) Isoprene
(III) Buna-N	(c) Chloroprene
(IV) Buna-S	(d) Acrylonitrile

Which of the following combination is correct:

(1) (I) – (b) ; (II) – (c) ; (III) – (d), (IV) – (a)

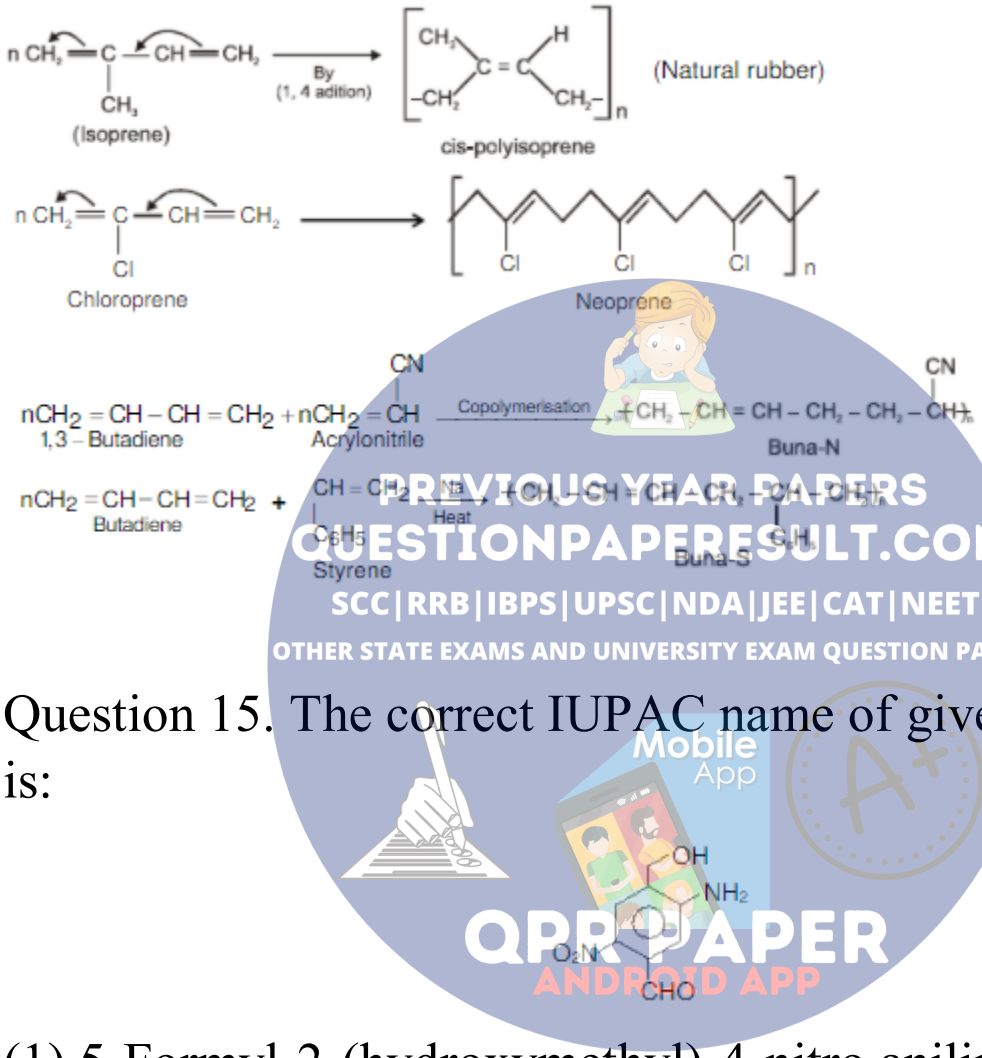
(2) (I) – (c) ; (II) – (b) ; (III) – (d), (IV) – (a)

(3) (I) – (b) ; (II) – (c) ; (III) – (a), (IV) – (d)

(4) (I) – (d) ; (II) – (b) ; (III) – (c), (IV) – (a)

Answer: (1)

Solution:



Question 15. The correct IUPAC name of given compound is:

(1) 5-Formyl-2-(hydroxymethyl)-4-nitro aniline

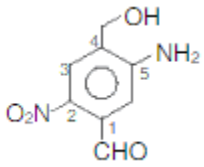
(2) 1-Amino-5-Formyl-2-(hydroxymethyl)-4-nitro benzene

(3) 3-Amino-4-(hydroxymethyl)-6-nitro benzene carbaldehyde

(4) 5-Amino-4-(hydroxymethyl)-2-nitro benzene carbaldehyde

Answer: (4)

Solution:



5-Amino-4-(hydroxymethyl)-2-nitro benzene carbaldehyde

Question 16.

Column – 1	Column – 2
(I) Benzene	(a) H_2-Pd-$BaSO_4$, Quinoline
(II) Benzonitrile	(b) CO, HCl/Anhydrous $AlCl_3$, CuCl
(III) Benzoyl chloride	(c) $SnCl_2 + HCl/H_3O^+$

Which combination gives benzaldehyde as a major product?

(1) (I) – (a) ; (II) – (c) ; (III) – (b)

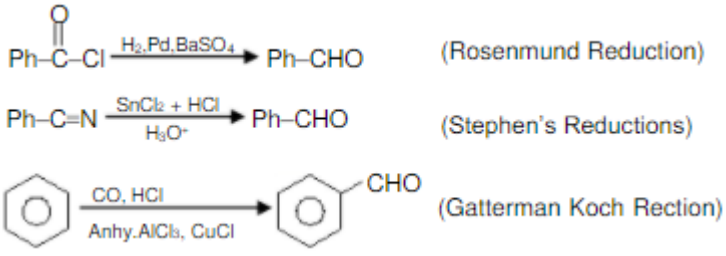
(2) (I) – (b) ; (II) – (c) ; (III) – (a)

(3) (I) – (c) ; (II) – (b) ; (III) – (a)

(4) (I) – (c) ; (II) – (a) ; (III) – (b)

Answer: (2)

Solution:



Question 17.

Column – 1	Column – 2
(I) Lucas reagent	(a) $C_6H_5SO_2Cl$/aq.NaOH
(II) Dumas method	(b) Conc. H_2SO_4/Δ
(III) Kjeldahl's method	(c) CuO/Δ
(IV) Heinsberg reagent	(d) An hy. $ZnCl_6$/Conc. HCl

Which of the following combination is correct:

(1) (I) – (b) ; (II) – (c) ; (III) – (d), (IV) – (a)

(2) (I) – (c) ; (II) – (b) ; (III) – (d), (IV) – (a)

(3) (I) – (d) ; (II) – (c) ; (III) – (b), (IV) – (a)

(4) (I) – (d) ; (II) – (b) ; (III) – (c), (IV) – (a)

Answer: (3)

Solution:

(I) Lucas reagent $\rightarrow$ Only $ZnCl_2$/Conc. HCl

(II) Dumas method $\rightarrow CuO/\Delta$

(III) Kjeldahl's method $\rightarrow$ Conc. H_2SO_4/Δ

(IV) Heinsberg reagent $\rightarrow C_6H_5SO_2Cl$ / aq.NaOH

Question 18. Which statement is incorrect regarding Lactose?

(1) It is composed of b-D(+)-Galactose & b-D(+)-Glucose

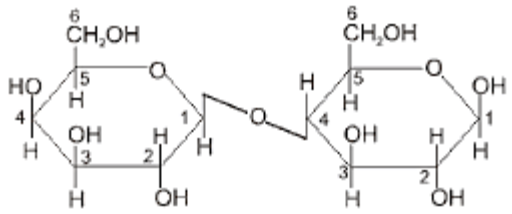
(2) It is a reducing sugar

(3) It is a disaccharides ($C_{12}H_{22}O_{11}$) having eight -OH groups

(4) It has glycosidic linkage of C1 of glucose and C_4 of galactose

Answer: (4)

Solution:



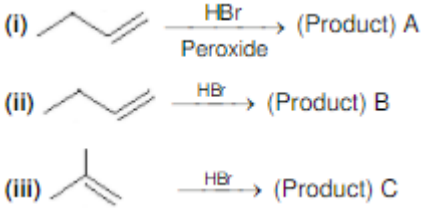
The linkage is between C – 1 of Galactose and C – 4 Glucose

Lactose (Milk sugar) $\xrightarrow{H_2O^{\oplus}}$ b-Galactose + b-Glucose

($C_{12}H_{22}O_{11}$)

It is hydrolysed by dilute acids or by the enzyme lactase, to an equimolecular mixture of D(+)-glucose and D(+)-galactose. Lactose is a reducing sugar.

Question 19. Complete the given reactions and compare the boiling points of their products?



(1) A > B > C

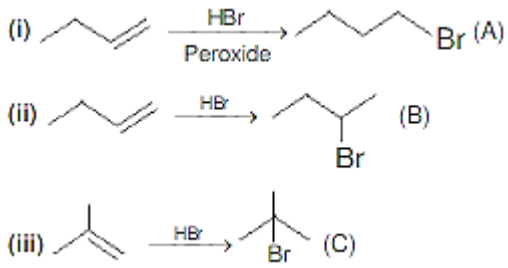
(2) A > C > B

(3) B > C > A

(4) C > B > A

Answer: (1)

Solution:

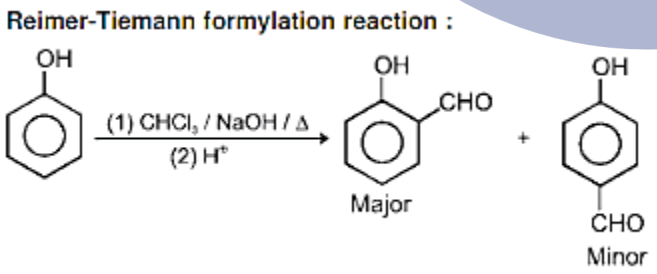


The boiling points of isomeric halo alkanes decrease with increase in branching.

Question 20. Complete the given reaction and find the percentage weight of carbon in the product.

Answer: 68.85

Solution:



Molecular formula of product is $C_7H_6O_2$

$$\text{Percentage weight of carbon} = \left(\frac{84}{122} \times 100\right) = 68.85\%$$

Question 21. For an equilibrium reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$; $K_c = 64$. What is the equilibrium constant the reaction $NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{2}{3}H_2(g)$

(1) $\frac{1}{8}$

(2) $\frac{1}{4}$

(3) $\frac{1}{64}$

(4) $\frac{1}{2}$

Answer: (1)

Solution:

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$K_c = \frac{[NH_3]^2}{[N_2][H_2]^3} = 64$$

For the reaction

$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{2}{3}H_2(g)$

$$K'_c = \frac{[NH_2]^{\frac{1}{2}}[H_2]^{\frac{3}{2}}}{[NH_3]} = \frac{1}{\sqrt{K_c}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$$

Question 22. Which of the following is the correct electronic configuration in octahedral complex with d_4 configuration?

(1) t_2g_4, eg_0 ; $\Delta 0 < P$

(2) t_2g_3, eg_1 ; $\Delta 0 > P$

(3) t_2g_2, eg_2 ; $\Delta 0 < P$

(4) t_2g_3, eg_1 ; $\Delta 0 < P$

Answer: (4)

Solution:

For d_4 configuration if $\Delta 0 < P$ the electronic configuration is t_2g_3, eg_1

Question 23. Which of the following method is used to produce di hydrogen of high degree purity (> 99.95%) in maximum amount?

(1) Zn + NaOH

(2) with electrolysis of warm aqueous barium hydroxide solution between nickel electrodes

(3) by electrolysis of brine solution

(4) by electrolysis of alkaline water using platinum electrodes.

Answer: (2)

Solution:

Dihydrogen of high degree of purity (> 99.95%) is obtained by the electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

Question 24. In a metal oxide, oxide ions crystallises in ccp lattice in which metal M_1 occupies 50% of octahedral voids and metal M_2 occupies 12.5% of tetrahedral voids. Then the oxidation states of metal M_1 and M_2 respectively are

(1) +2, +4

(2) +1, +3

(3) +3, +2

(4) +3, +1

Answer: (3)

Solution:

In the ccp lattice of oxide ions effective number of O-2 ions

$$= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

In the ccp lattice,

No. of octahedral voids = 4

No. of tetrahedral voids = 8

Given M_1 metal atoms $= 4 \times \frac{50}{100} = 2$

Given M_2 metal atoms $= 8 \times \frac{12.5}{100} = 1$

\ Formula of the compound = $(M_1)_2(M_2)O_4$

\ Oxidation states of metals M_1 & M_2 respectively are +3 and +2.

Question 25. For the given concentration cell Cu(s) | Cu^{2+} (C_2M) || Cu^{2+} (C_1M) | Cu(s)

Gibb's energy (ΔG) is negative if:

(1) $C_2 = \frac{C_1}{\sqrt{2}}$

(2) $C_2 = 2C_1$

(3) $C_2 = \sqrt{2}C_1$

(4) $C_1 = C_2$

Answer: (2)

Solution:

For concentration cell Eocell = 0

Anode: $Cu(s) \rightarrow Cu^{2+}(aq)A$

Cathode: $\underline{Cu^{2+}(aq)C} \rightarrow \underline{Cu(s)}$

Overall: $Cu^{2+}(aq)C \rightarrow Cu^{2+}(aq)A$

As $\Delta G = -nF\ E_{cell}$

If ΔG = -ve then E_{cell} is positive

$$E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{C_2}{C_1}$$

$$E_{cell} = \frac{-0.059}{2}\log\frac{C_2}{C_1}$$

$$E_{cell} > 0 \Rightarrow C_2 < C_1$$

Question 26. Main constituent part of misch metal alloy is:

(1) Lanthanids

(2) Actinides

(3) Transition metal

(4) Both by Lanthanides and Actinides

Answer: (1)

Solution:

Misch metal consists of Lanthanide metal (≈95%) and iron (≈5%) and traces of S, C, Ca and Al.

Question 27. Distillation process is used for purification of

(1) Zn

(2) Ge

(3) Ni

(4) Co

Answer: (1)

Solution:

Zn, Cd & Hg are purified by fractional distillation process.

Question 28. What is product of following reaction, NO + NO_2 →

(1) N_2O

(2) N_2O_3

(3) N_2O_4

(4) N_2O_5

Answer: (2)

Solution:

NO + NO_2 → N_2O_3

Question 29. In 180 gram water 10 gram each of A, B and C are mixed separately, then correct order of vapour pressure of these solution is

(Given Molar masses of A, B and C are 100, 150 & 125 gram/mole respectively)

(1) A > B > C

(2) B > C > A

(3) A > C > B

(4) C > B > A

Answer: (2)

Solution:

Relative lowering in vapour pressure depends on no. of mole of solute greater the no. of mole of solute greater in RLVP and smaller will be vapour pressure.

So order of vapour pressure is B > C > A

Question 30. Average atomic mass of chlorine is 35.5 then the correct naturally occurring molar ratio of ^{35}Cl & ^{37}Cl is:

(1) 4 : 1

(2) 2 : 1

(3) 3 : 1

(4) 1 : 1

Answer: (3)

Solution:

	^{35}Cl	^{37}Cl
Molar ratio	x	1 – x

Mavg. = 35 × x + 37 (1 – x) = 35.5

35x + 37 – 37x = 35.5

2x = 1.5

$$x = \frac{3}{4}$$

So, ratio of ^{35}Cl : ^{37}Cl = 3 : 1

Question 32. Match the compounds listen in Column – I with use in Column – II

	Column – I		Column – II
(i)	$Ca(OH)_2$	(a)	White wash
(ii)	$CaSO_4$, $\frac{1}{2}$ H_2O	(b)	Antacid
(iii)	NaCl	(c)	Washing soda preparation
(iv)	$Mg(OH)_2$	(d)	In making moulds for plaster statues

(1) (i) – (b) ; (ii) – (d) ; (iii) – (c) ; (iv) – (a)

(2) (i) – (d) ; (ii) – (a) ; (iii) – (c) ; (iv) – (b)

(3) (i) – (a) ; (ii) – (d) ; (iii) – (c) ; (iv) – (b)

(4) (i) – (a) ; (ii) – (d) ; (iii) – (b) ; (iv) – (c)

Answer: (3)

Solution:

(1) $Ca(OH)_2$ is used in white wash.

(2) Plaster of paris used in making of molds for plaster statues

(3) NaCl is used in preparation of washing soda

(4) A suspension of $Mg(OH)_2$ in water is used in medicine as an antacid under name of milk of magnesia

Question 33. Sulphur on reaction with conc. H_2SO_4 gives 'X' which on reaction with NaOH gives 'Y', Y on further

reaction with dil. H_2SO_4 give ‘X’ again. Then ‘X’ and ‘Y’ are respectively:

(1) S, Na_2SO_3

(2) SO_2, Na_2SO_3

(3) SO_2, Na_2SO_4

(4) SO_3, Na_2SO_3

Answer: (2)

Solution:

$$S + \text{conc.}H_2SO_4 \rightarrow \underset{\text{'X'}}{SO_2} \xrightarrow{NaOH} \underset{\text{'Y'}}{Na_2SO_3} \xrightarrow{\text{dil.}H_2SO_4} \underset{\text{'X'}}{SO_2}$$

Question 34. What is the atomic number of an element whose IUPAC name is Unniluniium?

Answer: 101

Solution:

According to IUPAC convention for naming of elements with atomic number more than 100, different digits are written in order and at the end sum is added. For digits following naming is used

0-nil

1-un

2-bi

3-tri

and so on…

Mathematic

Question 35. If α, β are the roots of equation $2x(2x + 1) = 1$ then β= ?

(1) α^2

(2) $-2\alpha(\alpha+1)$

(3) $2\alpha(\alpha+1)$

(4) $\alpha(\alpha-1)$

Ans. (2)

Sol. Given equation is $2x(2x + 1) = 1 \Rightarrow 4x2 + 2x - 1 = 0$ ..(1)

roots of equation (1) are α and β

$\therefore \alpha+\beta=-\frac{1}{2} \quad \Rightarrow \beta=-\frac{1}{2}-\alpha$ ....(2)

and $4\alpha^2+2\alpha-1=0 \quad \Rightarrow \alpha^2=\frac{1}{4}-\frac{\alpha}{2}$ .....(3)

Now, $-2\alpha(\alpha+1)=-2\alpha^2-2\alpha$

$=-2\left(\frac{1}{4}-\frac{\alpha}{2}\right)-2\alpha=-\frac{1}{2}-\alpha=\beta$.

Question 36. A plane intersects the x, y, z axis at A, B, C respectively. If G(1,1,2) is centroid of ΔABC, then the equation of the line perpendicular to plane and passing through G is

(1) $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-2}{2}$

(2) $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-2}{2}$

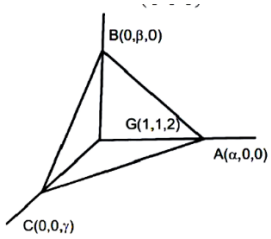
(3) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

(4) $\frac{x-1}{3}=\frac{y-1}{3}=\frac{z-2}{6}$

Ans. (3)

Sol. Let d $A(\alpha,0,0)$, $B(0,\beta,0)$, $C(0,0,\lambda)$ then

$G\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\lambda}{3}\right)\equiv(1,1,2)$



$\alpha=3,\beta=3,\lambda=6$

$\therefore$ equation of plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\lambda}=1$

$$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1$$

$$\Rightarrow 2x+2y+z=6$$

$\therefore$ required line $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

Question 37. Total number of words (with or without meaning) from letters of word 'LETTER' if no two vowels are together

(1) 100

(2) 110

(3) 120

(4) 180

Ans. (3)

Sol. Consonants are L, T, T, R

Vowels are E, E,

Total number of word (with or without meaning) from letters of word 'LETTER' $=\frac{6!}{2!2!}=180$

Total number of words (with or without meaning) from letters of word 'LETTER' if vowels are together $=\frac{5!}{2!}=60$

$\therefore$ Required = 180 – 60 = 120

Question 38. If the constant terms in the expansion of $\left(\sqrt{x}-\frac{K}{x^2}\right)^{10}$ is 405 then |k|= ?

(1) 9

(2) $\frac{9}{2}$

(3) 3

(4) 6

Ans. (3)

Sol. $T_{r+1} = {}^{10}C_r.\left(\frac{-K}{x^2}\right)^r\left(\sqrt{x}\right)^{10-r}$

$= {}^{10}C_r.(-K)^r.x^{5-\frac{5r}{2}}$

For constant term $\Rightarrow 5-\frac{5r}{2}=0 \Rightarrow r=2$

$\Rightarrow T_3 = {}^{10}C_2.K^2 = 405$

$\Rightarrow \frac{10(9)}{2}k^2 = 405$

$\Rightarrow K^2 = 9 \Rightarrow |K| = 3$

Question 39. Centre of a circle passing through point (0, 1) and touching the curve $y = x^2$ at $(2,4)$ is

(1) $\left(\frac{16}{5},\frac{53}{10}\right)$

(2) $\left(\frac{-16}{5},\frac{53}{10}\right)$

(3) $\left(\frac{-16}{5},\frac{-53}{10}\right)$

(4) $\left(\frac{16}{5},\frac{-53}{10}\right)$

Ans. (2)

Sol. $y = x^2$, (2, 4)

Tangent at (2, 4) is

$$\frac{1}{2}(y+4) = 2x$$

$y + 4 = 4x \Rightarrow 4x - y - 4 = 0$

Equation of circle $(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$

It passes through (0, 1)

$\therefore 4 + 9 + \lambda(0 - 1 - 4) = 0$

$13 = 5\lambda \Rightarrow \lambda = \frac{13}{5}$

$\therefore$ circle is $x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{13}{5}(4x - y - 4) = 0$

$$\Rightarrow x^2+y^2+\left(\frac{52}{5}-4\right)x-\left(8+\frac{13}{5}\right)y+20-\frac{52}{5}=0$$

$$\Rightarrow x^2+y^2+\frac{32}{5}x-\frac{53}{5}y+\frac{48}{5}=0$$

$\because$ centre is $\left(-\frac{16}{5},\frac{53}{10}\right)$

Question 40. If $\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $\beta = A^4 + A$ then determinant of $\beta = ?$

(1) $2-2.\cos 3\theta$

(2) $2+3.\cos 2\theta$

(3) $3+\cos 2\theta$

(4) $2+2.\cos 3\theta$

Ans. (4)

Sol. $A^2 = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$$A^2 = \begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$$

$$A^4 = \begin{bmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{bmatrix}$$

$$\beta = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -(\sin 4\theta + \sin\theta) & \cos 4\theta + \cos\theta \end{bmatrix}$$

$$|\beta| = (\cos 4\theta + \cos\theta)^2 + (\sin 4\theta + \sin\theta)^2$$

$$= 2 + 2(\cos 4\theta.\cos\theta + \sin 4\theta.\sin\theta)$$

$$= 2 + 2\cos(4\theta - \theta)$$

$$= 2 + 2.\cos 3\theta.$$

Question 41. Contrapositive of "If $n^3 - 1$ is even then n is odd" is

(1) If n is odd then $n^3 - 1$ is not even

(2) If n is not odd then $n^3 - 1$ is even

(3) If n is not odd then $n^3 - 1$ is not even

(4) If n is odd then $n^3 - 1$ is even

Ans. (3)

Sol. p = $n^3 - 1$ is even, q : n is odd

Contrapositive of $p \rightarrow q = \sim q \rightarrow \sim p$

$\Rightarrow$ "If n is not odd then n3 – 1 is not even"

Question 42. If $a_1, a_2, a_3 \ldots\ldots a_n$ and $b_1, b_2, b_3, \ldots\ldots b_n$ are two arithmetic progression with common difference of 2nd

is two more than that of first and b100 = a70, a100 = -399, a40 = -159 then the value of b1 is

(1) -51

(2) -61

(3) -81

(4) 81

Ans. (3)

Sol. Let a1, a1 + d, a1 + 2d ….first A. P.

a40 = a1 + 39d = -159 …(1)

a100 = a1 + 99d = -399 …(2)

from equation (1) and (2)

d = -4, a1 = -3

Now, b100 = a70

$\Rightarrow b_1 + 99D = a_1 + 69d$

$b_1 + 99 \times (-2) = -3 + 69 \times (-4)$(According to question D = d + 2)

$\Rightarrow b_1 = -81.$

Question 43. If the angle of elevation of the top of a summit is 45° and a person climbs at an inclination of 30° upto 1km, where the angle of elevation of top becomes 60°, then height of the summit is

(1) $\frac{1}{\sqrt{3}+1}\text{km}$

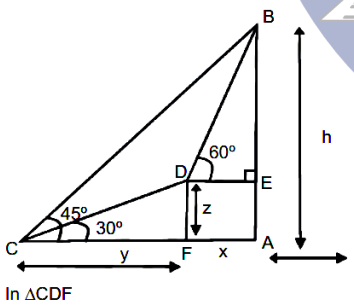
(2) $\frac{\sqrt{3}-2}{2}\text{km}$

(3) $\frac{\sqrt{3}+1}{2}\text{km}$

(4) $\frac{3}{\sqrt{3}-1}\text{km}$

Ans. (3)

Sol.



In ΔCDF

$$\sin 30^\circ = \frac{z}{1}\left[\text{CD} = 1\text{ km}(\text{given})\right]$$

$$z = \frac{1}{2} \qquad \ldots(1)$$

$$\cos 30^\circ = \frac{y}{1} \Rightarrow = \frac{\sqrt{3}}{2} \qquad \ldots(1)$$

Now in ΔABC

$$\tan 45^\circ = \frac{h}{x + y}$$

$$\Rightarrow h = x + y$$

$$\Rightarrow x = h - \frac{\sqrt{3}}{2} \qquad \ldots(2)$$

Now in ΔBDE

$$\tan 60^\circ = \frac{h - z}{x}$$

$$\sqrt{3}x = h - \frac{1}{2}$$

$$\Rightarrow \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2}$$

$$h = \frac{1}{\sqrt{3} - 1} = \frac{\sqrt{3} + 1}{2} \text{km}$$

Question 44. If ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots {}^nC_n$ are frequencies of n + 1 observations 1, 2, 22, …… 2_n such that mean is $\frac{729}{2^n}$ then value of n is:

Ans. (06.00)

Sol.

xi (observation)	1	2	22	… …	2n
fi (frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$	… …	${}^nC_n$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{1\times {}^nC_0 + 2\times {}^nC_1 + 2^n \times {}^nC_2 \ldots\ldots 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 \ldots\ldots + {}^nC_n}$$

$$= \frac{(1+2)^n}{(1+1)^n} = \frac{3^n}{2^n} = \frac{729}{2^n}$$

$$\Rightarrow 3^n = 3^6$$

$$\Rightarrow n = 6.$$

Question 45. Area bounded by curves $y = x^2 - 1$ and $y = 1 - x^2$ is

(1) $\frac{1}{3}$

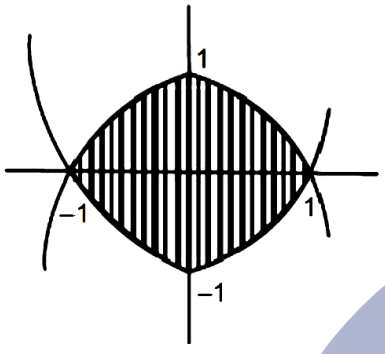
(2) $\frac{8}{3}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

Ans. (2)

Sol.



Given curves are $y = x^2 - 1$ and $y = 1 - x^2$

So, intersection points are $(\pm 1, 0)$ bounded area =

$$4\int_0^1 (1 - x^2)\,dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = 4\left(1 - \frac{1}{3}\right) = \frac{8}{3} \text{ sq. units}$$