# JEE MAINS 2020 2nd SEPT SHIFT 2

## Physics

Question 1. A rod is heated from O to 10°C its length is changed by 0.02% by what % mass density will change?

(1) 0.02

(2) 0.04

(3) 0.06

(4) 0.08

Ans. (3)

Sol. $\Delta\ell = \ell\alpha\Delta t$

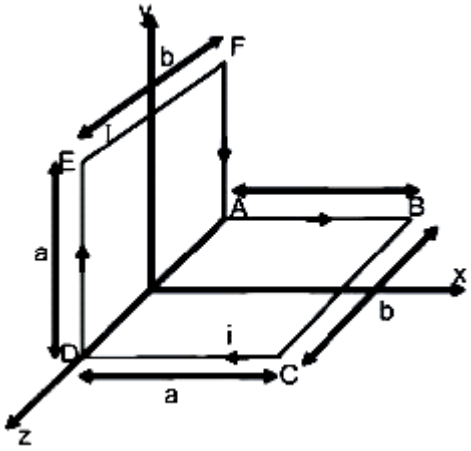
$$\alpha = \frac{\Delta\ell}{\ell\Delta T} = \frac{0.02}{100\times10}; \alpha = 2\times10^{-5}$$

$$\eta = 3\alpha = 6\times10^{-5}$$

$$\frac{\Delta V}{V} = \eta\times\Delta T; \frac{\Delta V}{V}\times100 = \left(6\times10^{-5}\times10\times100\right) = 6\times10^{-2}$$

Volume increase by 0.06% therefore density decrease by 0.06%.

Question 2.




Find magnetic moment of the loop

(1) $iab\left(\hat{i}+\hat{j}\right)$

(2) $iab\left(-\hat{i}-\hat{j}\right)$

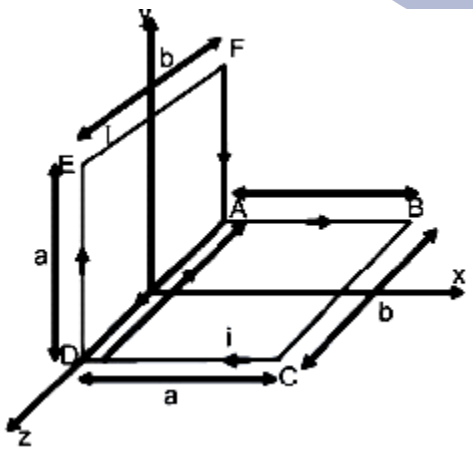
(3) $iab\left(\hat{i}+\hat{k}\right)$

(4) $iab+\left(\hat{j}+\hat{k}\right)$



Ans. (2)

Sol.




Loop ABCD

$$\vec{M}_1 = (abi)(-\hat{j})$$

For Loop DEFA

$$\vec{M}_2 = abi(-\hat{i})$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 ; \vec{M} = abi(-\hat{i} - \hat{j})$$

Question 3. If area (A), time (T) and momentum (P) is assume as fundamental quantities, then dimensional formula of energy will be:

(1) $AT^{-2}P^{1}$

(2) $A^{1/2}T^{-1}P^{2}$

(3) $A^{1/2}T^{-1}P^{1}$

(4) $AT^{-1/2}P^{2}$

Ans. (3)

Sol. Let dimension formula of energy will be

E = Aa Tb Pc

M1L2T-2=McL2a+c Tb-c

by comparison

C = 1 ... (1)

2a + c = 2 ... (2)

b - c = - 2 ... (3)

c = 1, a = 1/2, b = -1

E = A1/2 T-1 P1

Question 4. $\vec{E}$ & $\vec{B}$ in an electromagnetic wave oscillate along the direction having unit vectors $\hat{k}$ & $\hat{i}-\hat{j}$. Find unit vector along direction of propagation:

(1) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

(2) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(3) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$

(4) $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$



Ans. (1)

Sol. $\vec{E}\times\vec{B}\,\|\,\vec{C}$

$$\hat{E}\times\hat{B}=\frac{1}{\sqrt{2}}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&0&1\\1&-1&0\end{vmatrix}=\frac{\hat{i}-\hat{j}}{\sqrt{2}}\Rightarrow\hat{C}=\frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

Question 5. Charge Q is distributed on two concentric spheres of radius r and R respectively, if charge density of

both spheres is same then electric potential at the centre will be:

(1) $KQ\left(\frac{1}{r}+\frac{1}{R}\right)$

(2) $\frac{KQ(r+R)}{(r^2+R)}$

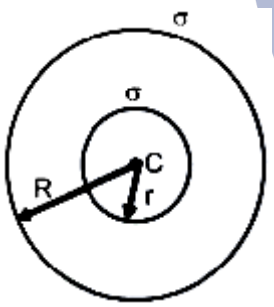
(3) $\frac{KQrR}{r+R}$

(4) $\frac{KQ(R^2+r^2)}{(R+r)}$



Ans. (2)

Sol.




$Q_1 = \sigma 4\pi r^2$

$Q_2 = \sigma 4\pi R^2$

$\therefore Q = \sigma 4\pi(r^2+R^2)$

$\therefore \sigma = \frac{Q}{4\pi(r^2+R^2)}$

$$V_C = \frac{KQ_1}{r} + \frac{KQ_2}{R}$$

$$= \frac{K\sigma 4\pi r^2}{r} + \frac{K\sigma 4\pi R^2}{R}$$

$$= K\sigma 4\pi (r + R)$$

$$= \frac{KQ4\pi (r + R)}{4\pi (r^2 + R^2)}$$

$$= \frac{KQ(r + R)}{(r^2 + R^2)}$$



Question 6. A capillary of radius 0.15 mm is dipped in liquid of density $\rho = 667\ kg/m^3$. If surface tension of liquid is $\frac{1}{20} Nm^{-1}$ then find the height up to which liquid rises in capillary. Angle of contact between liquid and capillary tube is 60°. (g = 10m/s2)

(1) 0.01 m

(2) 0.02 m

(3) 0.04 m

(4) 0.05 m

Ans. (4)

Sol. $h = \frac{2T\cos\theta}{\rho g r}$ $\theta = 60°$

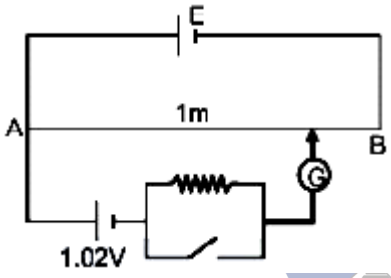
$$\rho = 667\ kg/m^3;\ T = 1/20\ Nm^{-1};\ r = 0.15\times10^{-3}$$

$$= \frac{2\times\frac{1}{20}\times\frac{1}{2}}{667\times10\times0.15\times10^{-3}}$$

$$= \frac{1000\times100}{20\times10\times667\times15}$$

$$= 0.05\ m$$

Question 7. In given potentiometer circuit 1.02 volt is balanced at 51 cm from A. Find potential gradient of potentiometer wire AB:






(1) 0.01 volt/cm

(2) 0.2 volt/cm

(3) 0.3 volt/cm

(4) 0.4 volt/cm

Ans. (2)

Sol. $X = V/\ell$

$$x = \frac{1.02}{51}$$

= 0.2 volt/cm

Question 8. In hydrogen atom electron jumps from (n + 1)th state to nth state the frequency of emitted photon is directly proportional to (n >> 1)

(1) n

(2) $\frac{1}{n}$

(3) $\frac{1}{n^2}$

(4) $\frac{1}{n^3}$



Ans. (4)

Sol. $E_n = -\frac{Rhc}{n^2}$

$E_{n+1} = -\frac{Rhc}{(n+1)^2}$

$\Delta E = E_{n+1} - E_n$

$hv = Rhc\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$

$v = R.c\left[\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right]$

$$v = R.c\left[\frac{1+2n}{n^2(n+1)^2}\right]$$

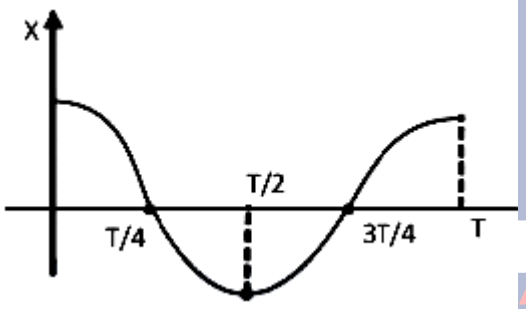
$$n >> 1 \Rightarrow v = R.c\left[\frac{2n}{n^2 \times n^2}\right]$$

$$= \frac{2RC}{n^3}$$

$$v \propto \frac{1}{n^3}$$



Question 9. Displacement time graph of particle performing SHM is as shown in figure. Assume that mean position is at x = 0




(A) No force acting on the particle at $\frac{T}{4}$

(B) Speed of particle is maximum at $\frac{3T}{4}$

(C) Acceleration at $\frac{T}{4}$ is maximum

(D) KE and PE is equal at $t = \frac{T}{8}$

(1) A & C

(2) A, C, D

(3) A, B, D

(4) C, D

Ans. (3)

Sol. From graph equation of SHM.

$X = A\cos\omega t$

(i) at $\frac{T}{4}$ particle at mean position

$\therefore a = 0$

$F = 0$

(ii) at $\frac{3T}{4}$ particle again at mean position so velocity is maximum

(iii) at $t = \frac{T}{4}$, particle is at mean position.

$\therefore a = 0$

(iv) KE = PE

$$\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

$$A^2 = 2x^2$$

$$x = \frac{+A}{\sqrt{2}}$$

$$\frac{A}{\sqrt{2}} = A\cos\omega t$$

$$t = \frac{T}{8}$$

$\therefore$ A, B and D are correct.

Question 10. A closed box contains an ideal gas if temperature of gas increased which:

Which of the following is correct.

(A) Mean free path remain same

(B) Mean free path decreases

(C) Relaxation time decreases

(D) Relaxation time remain same

(1) B & D

(2) A & C

(3) A & D

(4) B & C

Ans. (2)

Sol. As we know mean free path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

N = no. of molecule

V = volume of container

d = diameter of molecule

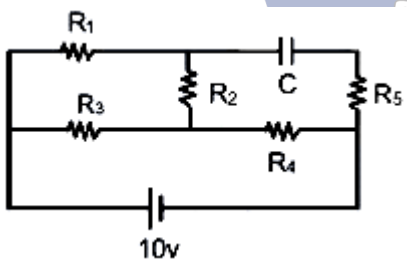
Velocity constant and no. of molecules are same.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decrease.



Question 11. $R_1 = R_2 = R_3 = R_4 = R_5$. Find voltage across capacitor at steady state




(1) 0 V

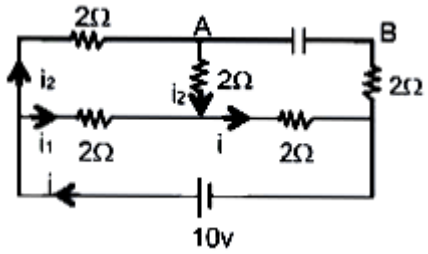
(2) 2V

(3) 4V

(4) 8V

Ans. (4)

Sol.




$$i = \frac{10}{\frac{4}{3}+2} = \frac{10\times 3}{10} = 3\text{Amp}$$

$$i_1 = 2A \;\&\; i_2 = 1A$$

$$VAB = 1\times 2 + 3\times 2 = 8V$$

Question 12. Two disc heaving moment of inertias l1 and l2 and angular velocities $\omega_1$ & $\omega_2$ are placed coaxially find total kinetic energy when they rotates with same angular velocity.

$l_1 = 0.10\left(kg-m^2\right)$ $\quad l_2 = 0.20\left(kg-m^2\right)$

$\omega_1 = 10\ Rad/sec$ $\quad \omega_2 = 5\ Rad/sec$

(1) 0 J

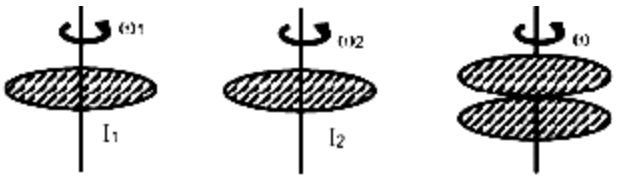
(2) 5 J

(3) 10 J

(4) 20/3 J

Ans. (4)

Sol.




$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{20}{3}$$

Final K.E.

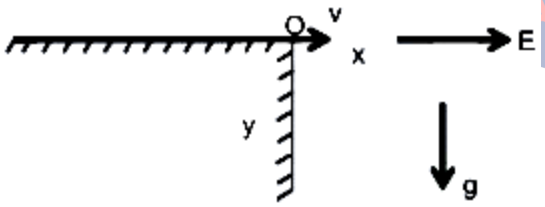
$$K_f = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2$$

$$= \frac{1}{2}(0.1+0.2)\times\left(\frac{20}{3}\right)^2$$

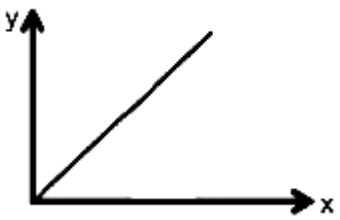
$$\Rightarrow k_f = \frac{20}{3}$$



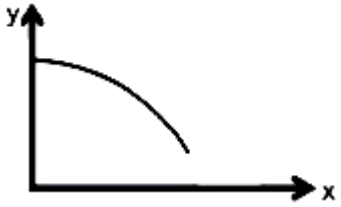
Question 13. A particle having mass m & charge + q is projected horizontally from point O. Choose correct option representing motion of the charge.




(1)



(2)



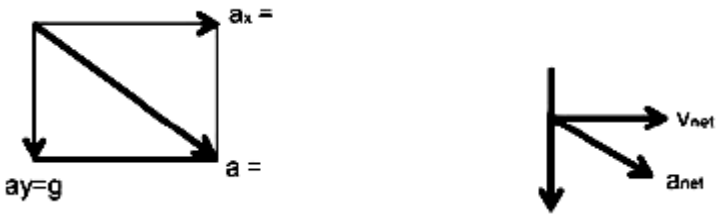

(3)


(4)

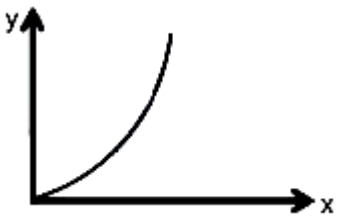



Ans. (3)

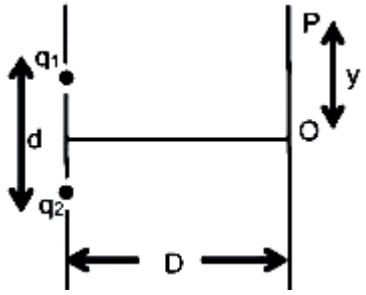
Sol.




Net acceleration of particle is constant therefore path is parabola



Question 14. In YDSE when $\lambda = 700$ nm then total number of fringes b/w O & P is 16. When $\lambda = 400$nm then total number of fringes b/w O & P is




(1) 14

(2) 28

(3) 7

(4) 12

Ans. (2)

Sol. $y = \dfrac{m_1 D\lambda_1}{d} = \dfrac{mD\lambda_2}{d}$

$$\frac{m_2}{m_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow m_2 = \frac{700}{400} \times 16 = 28$$



Question 15. Acceleration due to gravity is same at height h from surface and at the depth h from the surface, then find the value of h.

(1) $\left(\sqrt{5}-1\right)\dfrac{R}{2}$

(2) $\frac{\sqrt{5}R}{2}-1$

(3) $\frac{R}{\sqrt{2}}$

(4) $\frac{\sqrt{5}R+R}{2}$

Ans. (2)

Sol. $\frac{GM}{(R+h)^2}=\frac{GM}{R^3}(R-h)$

$R3 = (R + h)2 (R - h)$

$= (R2 + h2 + 2hR) (R - h)$

$R3 = R3 + h2R + 2hR2 - R2h - h3 - 2h2R$

$h3 + h2 (2R - R) - R2h = 0$

$h3 + h2R - R2= 0$

$h2 + hR - R2 = 0$

$h=\frac{-R\pm\sqrt{R^2+4(1)R^2}}{2}$

$=\frac{-R+\sqrt{5}R}{2}$

$=\frac{(\sqrt{5}-1)}{2}R$

Question 16. Efficiency of cyclic process is 50% if heat $Q_1$ = 1915J, $Q_2$ = 40J, $Q_3$ = 125J, then $Q_4$ is unknown then find the value of $Q_4$.

(1) 1080

(2) -980 J

(3) -1080

(4) -1280 J

Ans. (2)

Sol. $\eta = \dfrac{W}{\Sigma Q_+} = \dfrac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$

$\Rightarrow \dfrac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$

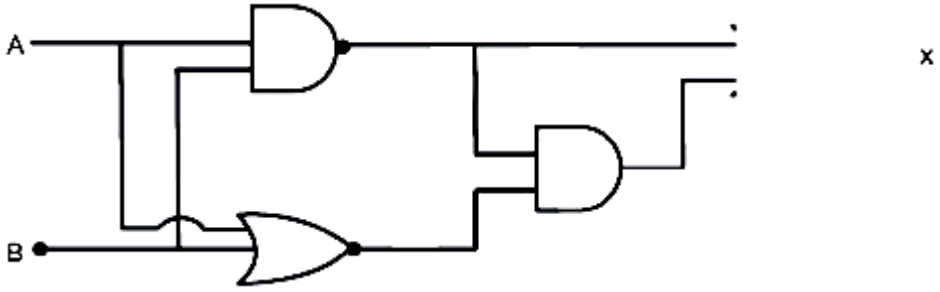
$\Rightarrow 1915 - 40 + 125 + Q_4 = 1020$

$\Rightarrow Q_4 = 1020 - 2000$

$\Rightarrow Q_4 = -980$ J



Question 17.




(A, B) is (0, 0), (0, 1), (1, 0), (1, 1) then output at x will be

(1) (0, 0, 0, 1)

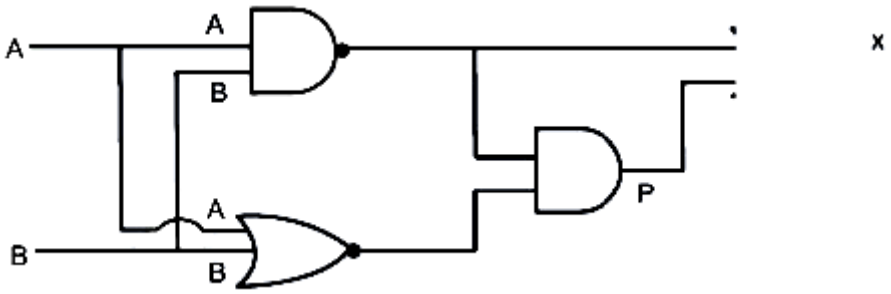
(2) (1, 1, 1, 0)

(3) (1, 0, 1, 0)

(4) (1, 1, 1, 1)

Ans. (1)

Sol.




| A | B | $\overline{A.B}$ | $A+B$ | $P=(\overline{AB}).(A+B)$ | $Q=P+(\overline{AB}$ | $\overline{Q}=X$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 |

$\therefore$ option (1) is the Answer.

Question 18. Impedance of L - R circuit is 100 Ωand phase difference between source voltage and source current is 45° of frequency if source 1000 Hz then find Inductance of coil.

(1) $25\sqrt{2}$ mH

(2) $\frac{50\sqrt{2}}{\pi}$ mH

(3) $\frac{25\sqrt{2}}{\pi}$ mH

(4) $\frac{20\sqrt{2}}{\pi}$ mH

Ans. (3)

Sol. $\tan\theta = \frac{X_L}{R} = \tan 45°$

$x_L = R$

$= 100 = \sqrt{x_L^2 + R^2}$

$100 = \sqrt{R^2 + R^2}$

$\sqrt{2}R = 100$

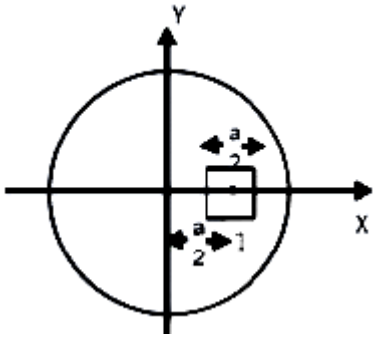
$R = 50\sqrt{2}$

$\therefore X_L = 50\sqrt{2}$

$L\omega = 50\sqrt{2}$

$L = \frac{50\sqrt{2}}{2\pi \times 1000} = \frac{25\sqrt{2}}{\pi}$ mH.

Question 19. A square of side $\frac{a}{2}$ is removed from a disc having radius a. Find centre of mass of remaining portion.




(1) $x = \frac{-2a}{\pi}$

(2) $\frac{-a}{8\pi - 2}$

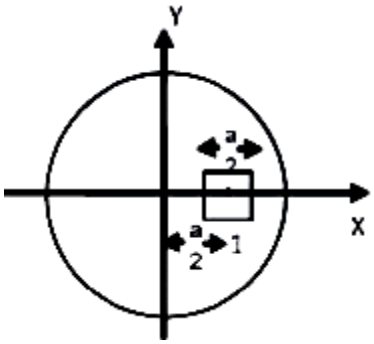
(3) $\frac{-4a}{3\pi}$

(4) $\frac{-a}{3\pi - 4}$



Ans. (2)

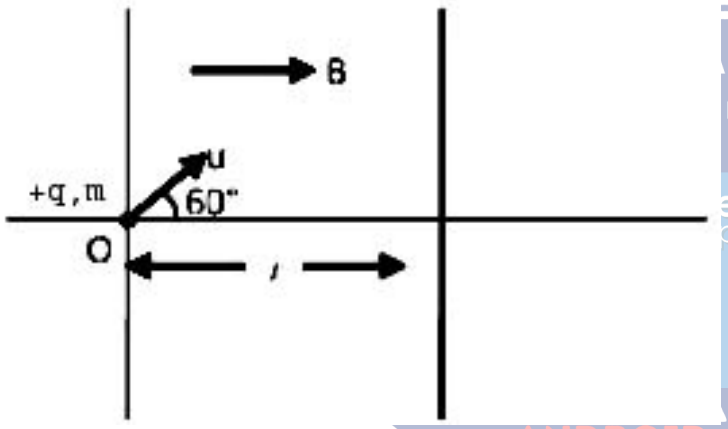
Sol.



$$X_a = \frac{A.x - A.x1}{A - A_1} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-a^3/8}{\left(\pi - \frac{1}{4}\right)a^2} = \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2}$$

Question 20. A particle is projected with velocity v from point O if particle makes 10 revolutions before coming out. Find length l traveled by particle before coming out




(1) $\frac{\pi mv}{qB}$

(2) $\frac{2\pi mv}{qB}$

(3) $\frac{20\pi mv}{qB}$

(4) $\frac{40\pi mv}{qB}$

Ans. (3)

Sol. $l = 10 \times \text{pitch}$

$$= 10 \times v\cos 60° \times \frac{2\pi m}{qB}$$

$$l = \frac{20\pi mv}{qB}$$

Question 21. Young modulus of a string of length 1m and density 900 kg/m3 is 9×109 N/m2. Find minimum resonance frequency (in Hz) can be produced in the string if strain in the string is 4.9 × 10-4.

Ans. (35.00 Hz)

Sol. Fundamental frequency in the string

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{\rho A}} = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{\rho l}}$$

$$f = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{\rho l}}$$

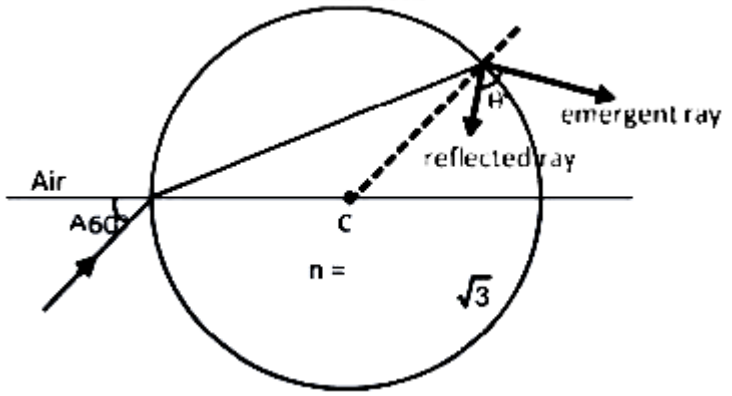
$$\left(\frac{\Delta l}{l} = 4.9\times10^{-4}\right)$$

$$= \frac{1}{2\times 1}\sqrt{\frac{9\times 10^{9}\times 4.9\times 10^{-4}}{900}}$$

$$= \frac{1}{2}\sqrt{49\times 10^{9-4-3}}$$

$$= \frac{1}{2}\times 70 = 35 \text{ Hz}$$

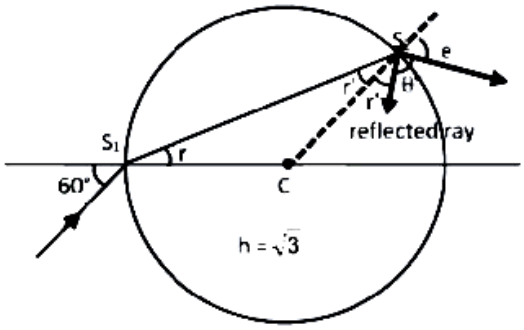
Question 22. Light incident on a sphere of refractive index $\sqrt{3}$ placed in a air as shown in figure. Find the angle $(\theta)$ in degree between emergent ray and reflected ray.






Ans. $(\theta = 90°)$

Sol.




Apply Snell's law at S1

$1 \sin 60° = \sqrt{3} \sin r$

$\sin r = \frac{1}{2}$

$r = 30°$

from geometry

$r' = 30°$

Again apply snell's law on S2

$\sqrt{3} \sin r' = 1 \sin e$

$\frac{\sqrt{3}}{2} = \sin e$

$\therefore e = 60°$

from geometry

$r' + \theta + e = 180°$

$\theta = 90°$



Question 23. A capacitor of capacity 20μF is charged up to 50V and disconnected from cell. Now this charged capacitor is connected to another capacitor of capacitance C. If final common potential is 20V then find the capacitance C.

Ans. (30.00)

Sol. $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$20 = \frac{20 \times 50 + 0}{20 + C}$$

$$400 + 20C = 1000$$

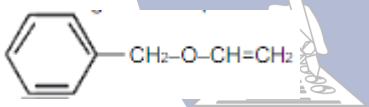
$$20C = 600$$

$$C = 30\mu F$$

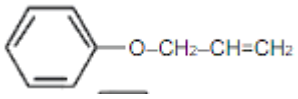
## Chemistry

Question 24. If given compound having molecular formula $C_9H_{10}O$ react with HI and produce two compounds A and B. A will give yellow ppt with $AgNO_3$ and B show positive iodoform test after tautomerization. Identify the structure of given compound.

(1)




(2)

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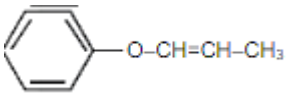


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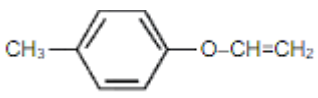
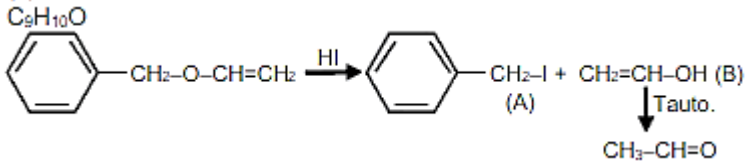
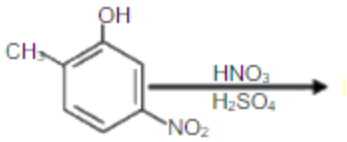



Answer: (a)

Solution:

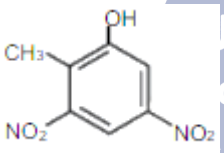



Question 25.

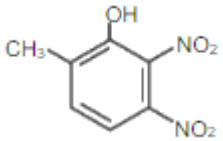



Product is

(1)




(2)

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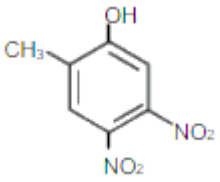
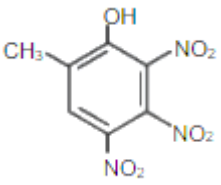


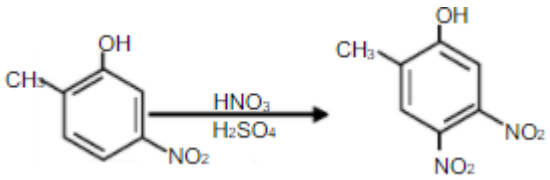

(4)



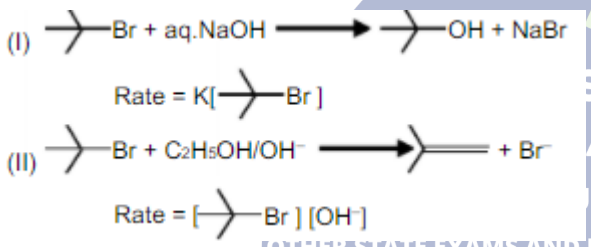
Answer: (c)

Solution:

This is electrophilic substitution reaction which is determine by electronic effect of OH\$CH_3$\$NO_2$.




Question 26.




Correct statement regarding there two reaction I and II.

(1) Rate of I reaction remain unchanged if concentration of OH- increases.

(2) Rate of II reaction remain unchanged if concentration of OH- increases.

(3) Rate of both reactions become double if concentration of OH- become double.

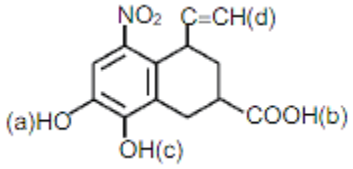
(4) Rate of both reactions do not depend upon concentration of OH-

Answer: (1)

Solution:

First reaction is $SN_1$ in which rate does not depend on conc. of nucleophile. Second reaction is $E_2$ reaction in which rate depends on conc. of base

Question 27. Find out order of acidic strength in following compound.




(1) a > b > c > d

(2) b > c > a > d

(3) b > c > d > d

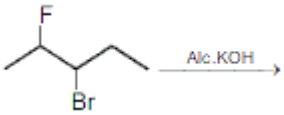
(4) b > d > a > c



Answer: (b)

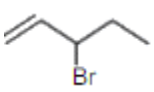
Solution: Carboxylic group is more acidic than phenol.

Question 28.

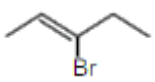



Product is:

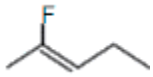
(1)



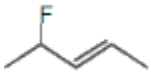

(2)




(3)

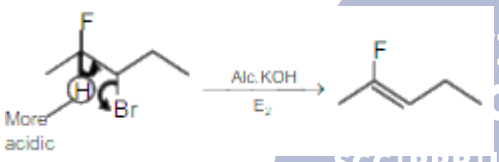



(4)




Answer: (c)

Solution:






Question 29. Sucrose $\xrightarrow{\text{Hydrolysis}}$ A + B $\xrightarrow[\text{Reagent}]{\text{Seliwanoff}}$

Which colour is obtained after above reaction?

(1) Red

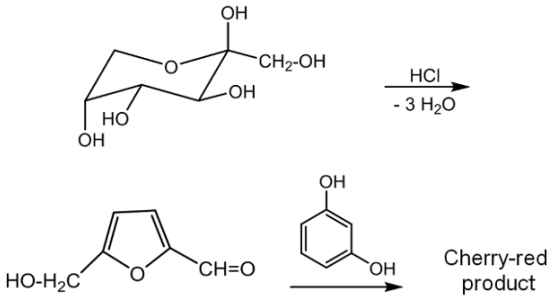
(2) Violet

(3) Blue

(4) Black

Answer: (1)

Solution:

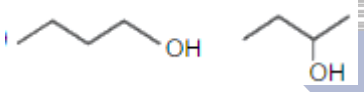



Question 30. Two acyclic compounds A & B having same molecular formula $C_3H_6O$. A and B react with $CH_3MgBr$ and give respectively C and D. C and D have following information.

| | C | D |
|---|---|---|
| Iodoform Test | -ve | +ve |
| Lucas Test | Instant +ve | after 5-minutes. |

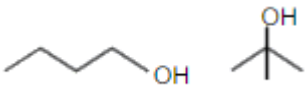
Identify structure of C and D are, respectively.

(1)

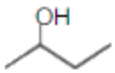


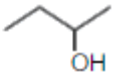

(2)


(3)



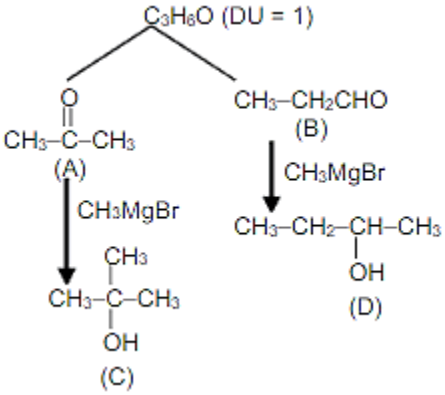

(4)




Answer: (2)

Solution:






Question 31. For the reaction $2A + B \rightarrow C$. Following experimental data are collected.

| Exp. No | $A\left[\frac{\text{Mole}}{\text{lit}}\right]$ | $B\left[\frac{\text{Mole}}{\text{lit}}\right]$ | Rate[mole/Lit sec] |
|---|---|---|---|
| 1 | 0.1 | 0.1 | $6\times10^{-3}$ |
| 2 | 0.2 | 0.1 | $1.2\times10^{-2}$ |
| 3 | 0.1 | 0.2 | $2.4\times10^{-2}$ |
| 4 | X | 0.2 | $7.2\times10^{-2}$ |
| 5 | 0.3 | Y | $2.88\times10^{-1}$ |

Find X and Y

(1) 0.2, 0.3

(2) 0.3, 0.4

(3) 0.4, 0.3

(4) 0.3, 0.2

Answer: (2)

Solution:

Rate = k[A]a[B]b

From Exp (1) & (2) a = 1

From Exp (1) & (2) a = 1

From Exp 3 & 4 $\Rightarrow$ $3 = \left(\frac{x}{0.1}\right)^2$ so x = 0.3

From Exp 1 & 5 $\Rightarrow 48 = (3)^1\left(\frac{y}{0.1}\right)^2$

$(4)^2 = \left(\frac{y}{0.1}\right)^2$ so y = 0.4



Question 32. No of subshells having n = 4 & m = -2 are:

(1) 2

(2) 4

(3) 8

(4) 16

Answer: (1)

Solution:

For n = 4 possible value of $\ell$ = 0,1,2,3 only $\ell$ = 2 & $\ell$ = 3 can have m = -2.

So possible subshells are 2.

Question 33. Which statement is correct when adsorption of gas take place on metal surface?

(A) ΔH becomes less negative with progress of reaction.

(B) With progress of reaction the strength of residual forces increases.

(C) $NH_3$ is absorbed more that $N_2$

(D) Equilibrium concentration of adsorbate increases with increase in temperature.

(1) A, C

(2) A, B

(3) B, D

(4) B, C



Answer: (1)

Solution:

(A) When gas is adsorbed on metal surface.

ΔH become less negative with progress or reaction.

(c) Gas with greater value of critical temperature (Tc) absorbed more. As $TC(NH_3) > TC(N_2)$

So $NH_3$ absorbed more than $N_2$

Question 34. Three element of 3rd period x, y, z such that the oxide of x is acidic, y is amphoteric and z is basic, the order of atomic no. of three elements is:

(1) x > y > z

(2) y > x > z

(3) z > x > y

(4) x > z > y

Answer: (1)

Solution:

On moving left to right in a period.

Acidic character of oxides is increase.

3rd period element oxides.

$\underbrace{Na_2O \quad MgO}_{\text{Basic}} \quad \underbrace{Al_2O_3}_{\text{Amphoteric}} \quad \underbrace{SiO_2 \quad P_2O_5 \quad Cl_2O_7}_{\text{Acidic}}$

(i) Acidic character ↑

(i) Atomic No ↑

So Z have minimum Atomic No & X have maxima Atomic No

So correct order is X > Y > Z

Question 35. Elements A and B do not form solid bicarbonate but reacts with $N_2$ to give nitrides. Which of the following can be A and B?

(1) Li, Mg

(2) Rb, Na

(3) Ca, Cs

(4) Ca, Na

Answer: (1)

Solution:

Li and Mg do not form solid bicarbonate. But react with N2 to give nitrides.

$$6Li + N_2 \xrightarrow{\Delta} 2Li_3N$$

$$3Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$$



Question 36. The structure of $SF_6$ is octahedral. What is the structure of $SF_4$ [including the lone pairs if any]?

(1) trigonal bipyramidal

(2) pyramidal

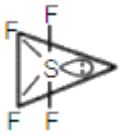
(3) square planar

(4) trigonal planar

Answer: (1)

Solution:

$SF_4$ ⇒ Steric No = 5 so hybridisation is sp3d




Structure is trigonal bipyramidal, but shape is “See Saw”

Question 37. Cast iron is used for the production of:

(1) Wrought iron, steel

(2) Wrought iron, pig iron, steel

(3) Pig iron, Wrought iron

(4) Pig iron, steel

Answer: (1)

Solution:

Cast iron is made from pig iron which is used for production of wrought iron & steel.



Question 38. Which of the following can’t show isomerism?

(1) $[Ni(NH_3)2Cl_2]$

(2) $[Ni(en)_3]$

(3) $[Pt(NH_3)2Cl_2]$

(4) $[Ni(NH_3)4(H_2O)_2]$

Answer: (1)

Solution:

(1) $\left[\overset{+2}{\mathrm{Ni}}\left(\mathrm{CH_3}\right)_2\mathrm{Cl_2}\right] \Rightarrow \mathrm{Ni}^{2+} \Rightarrow 3d^8 4s^0 \Rightarrow sp_3$

hybridisation ⇒ tetrahedral

So $[Ni(NH_3)2Cl_2]$ do not show isomerism.

(2) $[Ni(en)_3]$, show optical isomerism.

(3) $[Pt(NH_3)2Cl_2]$, show geometrical isomerism.

(4) $[Ni(NH_3)4(H_2O)_2]$, show geometrical isomerism.

Question 39. Structure of $XeF_5^-$ and $XeO_3F_2$ respectively are

(1) Pentagonal planar, trigonal bipyramidal

(2) Pentagonal planar, trigonal bipyramidal

(3) Trigonal bipyramidal, pentagonal planar

(4) Trigonal bipyramidal, trigonal bipyramidal

Answer: (1)

Solution:

(1) $XeF_5^-$ St. No = (5 + 2) = 7 so hybridization is = $sp_3d_3$ and structure is pentagonal planar.

(2) $XeO_3F_2$ St. No. = 5 so hybridization is = $sp_3d_3$ and structure is trigonal bipyramidal

Question 40. Match the following, proportionality with distance according to their interaction energy.

| | Species Interaction | | Interaction Energy Proportionality |
|---|---|---|---|
| (i) | ion-ion | a) | $\propto \frac{1}{r}$ |
| (ii) | Dipole – dipole | b) | $\propto \frac{1}{r^2}$ |
| (iii) | London dispersion | c) | $\propto \frac{1}{r^3}$ |
| | | d) | $\propto \frac{1}{r^6}$ |

Question 41. Which of the following is correct match?

(1) (i) – (a); (ii) – (c); (iii) – (d)

(2) (i) – (a); (ii) – (b); (iii) – (c)

(3) (i) – (b); (ii) – (c); (iii) – (d)

(4) (i) – (c); (ii) – (b); (iii) – (a)

Answer: (1)

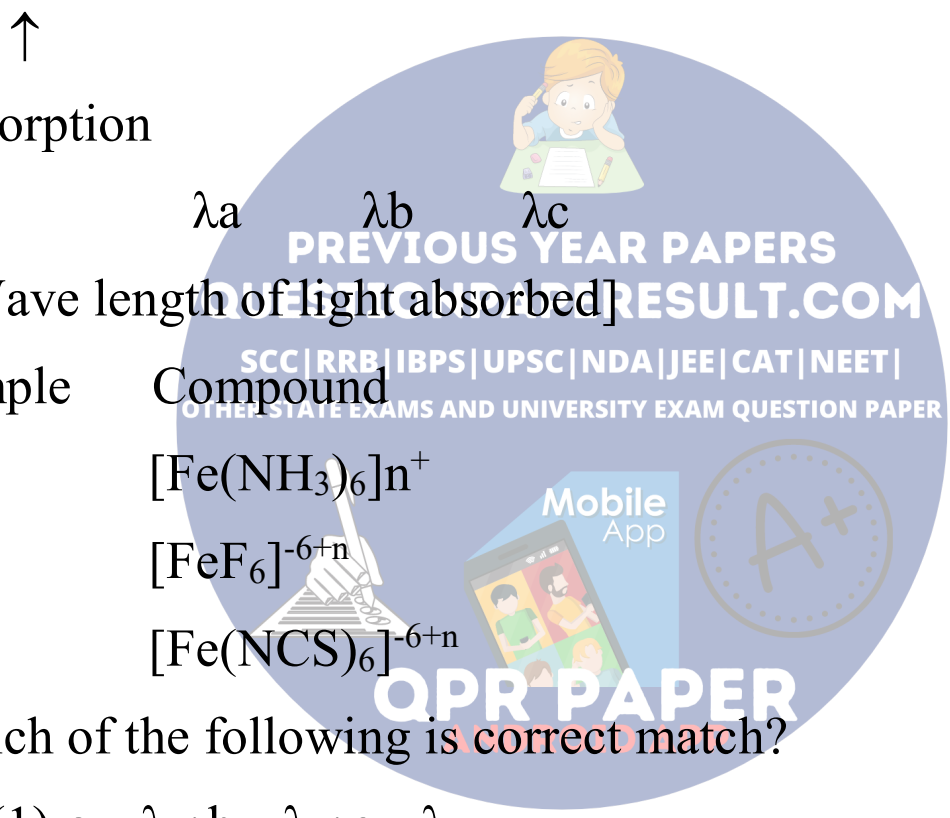
Solution:

(i) ion-ion interaction energy is inversely proportional to the distance between ions $\left(\frac{1}{r}\right)$

(ii) dipole-dipole interaction energy is inversely proportional to the third power of r $\left(\frac{1}{r^3}\right)$

(iii) The interaction energy of London force is inversely proportional to sixth power of distance between into interaction particles $\left(\frac{1}{r^6}\right)$

Question 42. The absorption spectra of three samples A, B, C is given




Sample Compound

(1) $[Fe(NH_3)_6]n^+$

(2) $[FeF_6]^{-6+n}$

(3) $[Fe(NCS)_6]^{-6+n}$

Which of the following is correct match?

(1) $a-\lambda_a; b-\lambda_b; c-\lambda_c$

(2) $a-\lambda_b; b-\lambda_c; c-\lambda_a$

(3) $a-\lambda_a; b-\lambda_c; c-\lambda_a$

(4) none of these

Answer: (3)

Solution:

Stronger the ligand greater is splitting of d orbitals and smaller will be wave length of light absorbed. The splitting power of ligands is $NH_3 > NC\overline{S} > F^-$

So order of wave length of light absorbed is $\lambda_{NH_3} < \lambda_{NC\overline{S}} < \lambda_{F^-}$

Question 43. If a mango shrinks when kept in concentrate salt solution, then which of the following process take place?

(1) diffusion

(2) dialysis

(3) osmosis

(4) reverse osmosis



Answer: (3)

Solution:

When mango kept in concentrate salt solution then solvent (water) flow from mango to concentrate solution that's why mango shrinks this called. "Osmosis"

Question 44. Heat of combustion of ethanol to give $CO_2$ and water at constant pressure and 27°C is -327 kcal. How much heat is evolved in (cal) in combustion at constant volume at 27°C?

Answer: 326400

Solution:

$$C_2H_5OH(\ell)+O_2(\ell)\rightarrow 2O_2(g)+3H_2O(\ell);\ \Delta H_C=-327\ \text{Kcal}$$

$$\Delta H_C=\Delta U_C+\Delta n_g RT$$

$$-327\times10^3=\Delta U_C+1\times2\times300$$

$$\Delta U_C=-326400\ \text{cal}$$

So heat evolved as constant volume is 326400 cal

Question 45. For cell reaction

$$2Cu^{+}\rightarrow Cu+Cu^{2+}$$

Find lnk = ……$\times10^{-1}$

Where k is equilibrium constant.

Given (i) $Cu^{+}+e\rightarrow Cu \quad E^{\circ}=0.52\,V$

(ii) $Cu^{2+}+E\rightarrow Cu^{+1} \quad E^{\circ}=0.16\,V$

$$\left(\frac{RT}{F}=0.025\right)$$



Answer: 144.00

Solution:

$$E^{\circ}_{cell}=E^{\circ}_{Cu^{+}/Cu}-E^{\circ}_{Cu^{2+}/Cu^{-1}}$$

$$=0.52-0.16$$

$$=0.36V$$

$$E^{\circ}_{cell}=\frac{RT}{nF}\ln K_{eq}$$

$$0.36 = \frac{0.025}{1} \ln k$$

$\ln k = 14.4$

$= 144 \times 10^{-1}$

Ans.144

Question 46. In a saturated acyclic compound the mass ratio of C:H is 4:1 and C:O is 3:4. Find the no. of moles of $O_2$ required to react with 2 moles compound to give $CO_2$ and water.

Answer: 5.00

Solution:

Mass ratio of C:H is 4:1 ⇒ 12:3

& C:O is 3:4 ⇒ 12:16

| | | Mass | mole | mole ratio |
|---|---|---|---|---|
| So | C | 12 | 1 | 1 |
| | H | 3 | 3 | 3 |
| | O | 16 | 1 | 1 |

Empirical formula ⇒ $CH_3O$

As compound is saturated a cyclic so molecular formula is $C_2H_6O_2$.

$$\underset{\text{mole}}{C_6H_6O_2} + \underset{5\text{ mole}}{\frac{5}{2}O_2(g)} \rightarrow 2CO_2(g) + 3H_2O(g)$$

So required moles of $O_2$ is $\Rightarrow 5$

Question 47. A metal having work function = $4.41 \times 10^{-19}$ J is subjected to a light having wavelength 300 nm, then maximum kinetic energy of the emitted. Photoelectron is ….. $\times 10^{-21}$ J.

(Given h = $6.63 \times 10^{-34}$ JS & C = $3 \times 10^{8}$ m/sec)

Answer: 222.00

Solution:

$E = E0 + (KE)_{max}$

$$\frac{hC}{\lambda} = 4.41\times10^{-19} + kE$$

$$\frac{6.63\times10^{-34}\times3\times108}{300\times10^{-9}} = 4.41\times10^{-19} + KE$$

$$So, (kE)_{max} = 6.63\times10^{-19} - 4.41\times10^{-19}$$

$= 2.22 \times 10^{-19}$

$= 222 \times 10^{-21}$J



Question 48. Let the oxidation state of the transition element of compound $K_2Cr_2O_7$, $KMnO_4$ and $K_2FeO_4$ be X, Y and Z respectively, calculate X+Y+Z

Answer: 19.00

Solution:

| Compound | Oxidation state of transition element. |
|---|---|
| (i) $K_2Cr_2O_7$ | X = +6 |
| (ii) $KMnO_4$ | Y = +7 |
| (iii) $K_2FeO_4$ | Z = +6 |

So (X + Y + Z) = 19

## Mathematics

Question 49. Find the imaginary part of

$$\left(\left(3+2\sqrt{-54}\right)^{1/2}-\left(3-2\sqrt{-54}\right)^{1/2}\right)$$

(1) $-\sqrt{6}$

(2) $-2\sqrt{6}$

(3) $\sqrt{6}$

(4) 6



Ans. (2)

Sol. $\left|3+2\sqrt{-54}\right| = \sqrt{9+216} = 15$

$$\Rightarrow \left(3+2\sqrt{-54}\right)^{1/2} = \pm\left(\sqrt{\frac{15+3}{2}}+i\sqrt{\frac{15-3}{2}}\right)$$

$$= \pm\left(3+i\sqrt{6}\right)$$

and $\left(3-2\sqrt{-54}\right)^{1/2} = \pm\left(3-i\sqrt{6}\right)$

Hence $\left\{\left(3+2\sqrt{-54}\right)^{1/2}-\left(3-2\sqrt{-54}\right)^{1/2}\right\}$

$=\pm 2i\sqrt{6}$ or $\pm 6$

Hence imaginary part $=\pm 2\sqrt{6}$.

Question 50. If a, b, c $\in$R such that a3 + b3 + c3 = 2 and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then find abc

(1) $\frac{2}{3}$

(2) $-\frac{2}{3}$

(3) $\frac{1}{3}$

(4) 1



Ans. (1)

Sol. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow abc = \frac{a^3+b^3+c^3}{3} = \frac{2}{3}$

Question 51. The ratio of three consecutive binomial coefficients in the expansion of (1 + x)n is 2:5:12 find n

(1) 120

(2) 34

(3) 118

(4) 35

Ans. (3)

Sol. ${}^{n}C_{r-1} : {}^{n}C_{r} : {}^{n}C_{r+1} = 2:5:12$

$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{5}{2}$ and $\Rightarrow \frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{12}{5}$

$\Rightarrow \frac{n-r+1}{r} = \frac{5}{2}$ and $\Rightarrow \frac{n-r}{r+1} = \frac{12}{5}$

$\Rightarrow 2n - 7r + 2 = 0$ and $5n - 17r - 12 = 0$

On solving n = 118 and r = 34



Question 52. There are n stations in a circular path. Two consecutive stations are connected by blue line and two non-consecutive stations are connected by red line. If no. of red lines is equal to 99 times number of blue line then value of n is

(1) 201

(2) 200

(3) 199

(4) 202

Ans. (1)

Sol. Two consecutive stations = n

Two non-consecutive stations = ${}^{n}C_2 - n$

$${}^{n}C_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} - n = 99n$$

$$\Rightarrow \frac{n^2 - n}{2} = 100n \Rightarrow n^2 = 201n \Rightarrow n = 201$$

Question 53. $\int_1^2 |2x - [3x]|\,dx = ?$ (where $[\cdot]$ denotes greatest integer function)

(1) 1

(2) 3

(3) 2

(4) 4

Ans. (1)

Sol. $\int_1^2 |3x - [3x] - x|\,dx$

$$\Rightarrow \int_1^2 |\{3x\} - x|\,dx = \int_1^2 (x - \{3x\})\,dx \Rightarrow \int_1^2 x\,dx - \int_1^2 \{3x\}\,dx$$

$$\Rightarrow \left(\frac{x^2}{2}\right)_1^2 - 3.\int_0^{1/3} 3x\,dx = \left(\frac{4}{2} - \frac{1}{2}\right) - 9\left(\frac{x^2}{2}\right)_0^{1/3} \Rightarrow \frac{3}{2} - \frac{9}{2}\left(\left(\frac{1}{3}\right)^2 - 0^2\right) = 1$$

Question 54. If $x^2 - y^2 \sec^2\theta = 10$ be a hyperbola and $x^2\sec^2\theta + y^2 = 5$ be an ellipse such that the eccentricity of hyperbola = $\sqrt{5}$ eccentricity of ellipse then find the length of latus rectum of ellipse

(1) $\frac{4\sqrt{5}}{3}$

(2) $\frac{4}{3\sqrt{5}}$

(3) $\frac{20\sqrt{5}}{3}$

(4) $\sqrt{30}$



Ans. (1)

Sol. $\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1 \Rightarrow e_H = \sqrt{1+\cos^2\theta}$ and

$\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \Rightarrow e_E = \sqrt{1-\cos^2\theta} = \sin\theta$

As given $e_H = \sqrt{5}\, e_E$

$\Rightarrow 1+\cos^2\theta = 5\sin^2\theta \Rightarrow \cos^2\theta = 2/3$

Now length of L.R. of ellipse $= \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

Question 55. $\lim_{x\to 0} \tan\left(\frac{\pi}{4}+x\right)^{1/x} = ?$

(1) e

(2) e2

(3) e4

(4) $\frac{1}{e}$

Ans. (2)

Sol. $e^{\lim_{x\to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)} \Rightarrow e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}} = e^{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)\left(\frac{2}{1-\tan x}\right)}$

$\Rightarrow e^{(1)\left(\frac{2}{1-0}\right)} = e^2$



Question 56. If f(x) be a quadratic polynomial such that f(x) = 0 has a root 3 and f(2) + f(-1) = 0 then other root lies in

(1) (-1,0)

(2) (0, 1)

(3) (-2, 1)

(4) (1,2)

Ans. (1)

Sol. Let f(x) = ax2 + bx + c

f(2) + f(-1) = 0 $\Rightarrow$ 5a + b + 2c = 0

and $f(3) = 0 \Rightarrow 9a + 3b + c = 0 \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$

product of roots $\alpha\beta = \frac{c}{a} = -\frac{6}{5}$

and $\alpha = 3 \Rightarrow \beta = -\frac{2}{5} \in (-1, 0)$

Question 57. If a curve $y = f(x)$ satisfy the differential equation $2x^2\, dy = (2xy + y^2)\, dx$ and passes (1,2) then find $f(1/2)$

(1) $\frac{1}{1+ln\,2}$

(2) $\frac{1}{1-ln\,2}$

(3) $\frac{2}{1-ln\,2}$

(4) $\frac{2}{1+ln\,2}$



Ans. (1)

Sol. $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \Rightarrow y^{-2}\frac{dy}{dx} - \frac{1}{y}.\frac{1}{x} = \frac{1}{2x^2}$

Put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} + \left(\frac{1}{x}\right)t = \frac{1}{2x^2}$

Linear differential equation

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{l n x} = x$$

So solution of the linear differential equation is

$$tx = \int \frac{1}{2x^2}.xdx + C \qquad \Rightarrow \frac{-x}{y} = \frac{1}{2} l n x + C$$

The curve passes through (1,2)

$$\Rightarrow \frac{1}{2} = \frac{1}{2} l n 1 + C \Rightarrow C = -\frac{1}{2}$$

$$\text{Hence} -\frac{x}{y} = \frac{1}{2} l n x - \frac{1}{2}$$



$$\text{or} \frac{x}{y} = \frac{1 - l n x}{2} \Rightarrow y = \frac{2x}{1 - l n x} \Rightarrow f\left(\frac{1}{2}\right) = \frac{2x\frac{1}{2}}{1 - l n \frac{1}{2}} = \frac{1}{1 + l n 2}$$

Question 58. If A, B, C are three pairwise independent events such that $P(A \cap B \cap C) = 0$ then $P((B^c \cap C^c)/A)$ is equal to -

(1) P(C) + P(B)

(2) P(CC) + P(B)

(3) P(BC) = P(C)

(4) P(CC) - P(B)

Ans. (4)

Sol. $P\left(\left(B^c \cap C^c\right)/A\right) = \dfrac{P\left(A \cap \left(B^c \cap C^c\right)\right)}{P(A)}$

$= \dfrac{P(A) - \left\{P\left(A \cap B\right) + P\left(A \cap C\right) - P\left(A \cap B \cap C\right)\right\}}{P\left(A\right)}$

$= \dfrac{P\left(A\right) - P\left(A\right).P\left(B\right) - P\left(A\right).P\left(C\right) + 0}{P\left(A\right)}$

$= 1 - P(B) - P(C)$

$= P(CC) - P(B)$ or $P(BC) - P(C)$



Question 59. If sum of series $(x + ka) + (x^2 + (k - 2)a) + (x^3 + (k - 4)a) + \ldots$ 9 terms is $\dfrac{x^{10} - x - 45a\left(x - 1\right)}{x - 1}$ then value of k is:

Ans. (3.00)

Sol. Given $(x + ka) + (x^2 + (k - 2)a) + (x^3 + (k - 4)a) \ldots$ 9 terms

$= \dfrac{x\left(x^9 - 1\right)}{x - 1} + \dfrac{9}{2}\left[2ka + \left(9 - 1\right)\left(-2a\right)\right]$

$= \dfrac{x^{10} - x}{x - 1} + \left(9ka - 72a\right) = \dfrac{x^{10} - x + 9a\left(k - 8\right)\left(x - 1\right)}{x - 1}$

$\therefore$ $9a\,(k - 8) = -45a$

$K - 8 = -5$

$K = 3$

Question 60. Point P divides line joining $A\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $B\left(2\hat{i}+\hat{j}+3\hat{k}\right)$ in the ration $\lambda:1$ such that $\overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$. Find $\lambda$.

Ans. (00.80)

Sol. P.V. of P is $\overrightarrow{OP}=\dfrac{\vec{a}+\lambda\vec{b}}{\lambda+1}$

$\because \overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$

$$\Rightarrow\vec{b}\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)-3\left|\vec{a}\times\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)\right|^2=6\Rightarrow\frac{\vec{a}.\vec{b}+\lambda\left|\vec{b}\right|^2}{\lambda+1}-\frac{3\lambda^2}{(\lambda+1)^2}\left|\vec{a}\times\vec{b}\right|^{-2}=$$

$$\Rightarrow\frac{6+\lambda.14}{\lambda+1}-\frac{3\lambda^2}{(\lambda+1)^2}.6=6\Rightarrow\frac{18\lambda^2}{(\lambda+1)^2}+6=6+\frac{8\lambda}{\lambda+1}$$

$$\Rightarrow 18\left(\frac{\lambda}{\lambda+1}\right)^2-\frac{8\lambda}{\lambda+1}=0\left(\frac{\lambda}{\lambda+1}\neq 0\right)$$

$$\Rightarrow 10\lambda=8\Rightarrow\ \lambda=0.8$$

Question 61. If $y=\alpha$ bisects the area bounded by region given by $x^2\le y\le 2x$ then:

(1) $8\alpha^{3/2}-3\alpha^2=8$

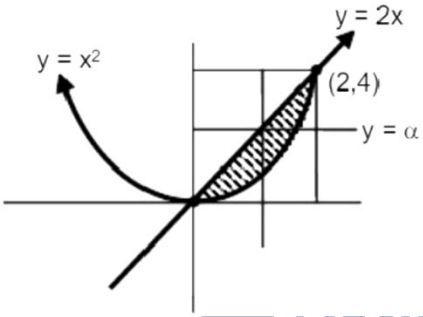
(2) $8\alpha^{3/2}-3\alpha^{2}=4$

(3) $4\alpha^{3/2}-3\alpha^{2}=8$

(4) $4\alpha^{3/2}+3\alpha^{2}=8$

Ans. (a)

Sol.




$$\int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy=2\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dx$$

$$\Rightarrow\left(\frac{2y^{3/2}}{3}-\frac{y^{2}}{2}\right)\Bigg|_0^4=2\left(\frac{2y^{3/2}}{3}-\frac{y^{2}}{4}\right)\Bigg|_0^{\alpha}\Rightarrow\frac{16}{3}-4=2\left(\frac{2}{3}a^{3/2}-\frac{\alpha^{2}}{4}\right)$$

$$\Rightarrow\frac{4}{3}=2\left(\frac{2}{3}\alpha^{3/2}-\frac{\alpha^{2}}{4}\right)\Rightarrow\frac{2}{3}=\frac{8\alpha^{3/2}-3\alpha^{2}}{12}$$

$$\Rightarrow 8\alpha^{3/2}-3\alpha^{2}=8$$

Question 62. If $\sin^{4}\theta+\cos^{4}\theta+\lambda=0$ has a real solution then range of $\lambda$ is

(1) [-1,1]

(2) $\left[-1,\frac{-1}{2}\right]$

(3) $\left[\frac{1}{2},1\right]$

(4) $\left[-\frac{1}{2},\frac{1}{2}\right]$

Ans. (2)

Sol. $-\lambda = \sin^4\theta + \cos^4\theta$

$= \left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - \frac{4\sin^2\theta\cos^2\theta}{2}$

$= 1 - \frac{\sin^2 2\theta}{2}$

$\lambda = \frac{\sin^2 2\theta}{2} - 1$

$\lambda \in \left[-1, -\frac{1}{2}\right]$



Question 63. If

$y = \sum_{k=1}^{6} k\cos^{-1}\left(\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right)$ then $\frac{dy}{dx} = ?$.

(1) 92

(2) 91

(3) 90

(4) 89

Ans. (2)

Sol. $y = \sum_{k=1}^{6} k\cos^{-1}\left(\cos kx.\cos\alpha - \sin kx.\sin\alpha\right)$

$= \Sigma\, k.\cos^{-1}\cos\left(kx+\alpha\right)$

$= \Sigma\, K\left(kx+\alpha\right) = \Sigma\left(k^2x + k\alpha\right)$

$$\frac{dy}{dx} = \sum_{k=1}^{6} k^2 = \frac{6(7)(13)}{6} = 91$$

Question 64. Let a1, a2, .... a11 are in increasing A.P. and if variance of these number is 90 then value of common difference of A.P. is.

Ans. (03.00)

Sol. Given a1, a2, a3,..... a11are in A.P.

$\therefore$ variance of (a1, a2, ..... a11)= 90

$$\Rightarrow \frac{\sum_{i=1}^{11} a_i^2}{11} - \left(\frac{\sum_{i=1}^{11} a_i}{11}\right)^2 = 90 \Rightarrow \frac{d^2}{10}.\frac{10.11.21}{6} - d^2.\frac{55}{11}.\frac{55}{11} = 90$$

$\Rightarrow$ 35d2 – 25d2 = 90 $\Rightarrow$ d2 = 9 $\Rightarrow$ d = 3

Question 65. An equilateral triangle is inscribed in parabola y2 = 8x whose one vertex coincide with vertex of parabola. Find area of triangle.

(1) $196\sqrt{3}$

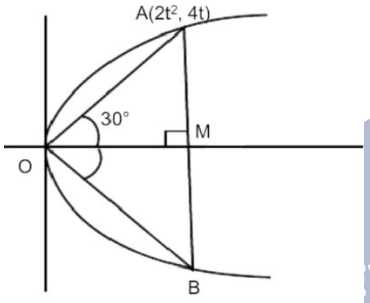
(2) $194\sqrt{3}$

(3) $192\sqrt{3}$

(4) $190\sqrt{3}$

Ans. (3)

Sol.




$y^2 = 8x,\ a = 2$

$A \equiv (2t2, 2(2)t) \equiv (2t2, 4t)$

$$\tan 30^\circ = \frac{4t}{2t^2} = \frac{2}{t} = \frac{1}{\sqrt{3}}$$

$t = 2\sqrt{3}$

Area of

$$\Delta OAB = 2.\Delta OMA = 2.\frac{1}{2}.(2t^2)(4t) = 8t^3 = 8\left(2\sqrt{3}\right)^3 = 192\sqrt{3}$$

Question 66. If $f(x) = \dfrac{\ln(1+x)}{x}, x \in (-1, \infty)$ and f(0) = 1

then f(x) is

(1) decreasing in (-1, 0) and increasing in $(0, \infty)$

(2) always increasing

(3) always decreasing

(4) increasing in $(-1,0)$ and decreasing in $(0,\infty)$

Ans. (3)

Sol. $F'(x)=\dfrac{\dfrac{x}{1+x}-\ln(1+x)}{x^2}=\dfrac{x-(1+x)\ln(1+x)}{x^2(1+x)}$

Let $g(x) = x - (1 + x)\ln(1 + x)$

$\Rightarrow g'(x) = 1 - 1 - \ln(1 + x)$

$= -\ln(1+x) \Rightarrow g'(x)=\begin{cases}>0 & \forall x\in(-1,0)\\ <0 & \forall x\in(0,\infty)\end{cases}$

$g_{max}$ at $x = 0 \Rightarrow g(0) = 0$

$g(x) < 0 \forall x \in (-1,\infty) \Rightarrow f'(x) < 0 \forall x \in (-1,\infty)$

f(x) decreasing $\forall x \in (-1,\infty)$.



Question 67. If $y = (1 + x)^2y + \cos2(\sin^{-1}x)$ be a curve then find equation of normal at x = 0.

(1) $x = 4y = 8$

(2) $x + 4y = 2$

(3) $2x + y = 2$

(4) $2x - y = 2$

Ans. (1)

Sol. $y = (1 + x)2y + \cos 2(\sin^{-1}x)$

$x = 0, y = 2$

$y = e^{2y\ln(1+x)} + (1 - x2)$

$$\frac{dy}{dx} = e^{2y\ln(1+x)}\left\{\frac{2y}{1+x} + \ln(1+x).2y'\right\} - 2x$$

$$y'\left(\frac{2\times 2}{1+0} + 0\right)$$

$Y' = 4$

$y - 2 = -1/4(x - 0)$

$4y - 8 = -x$

$x + 4y = 8$



Question 68. Which of the following is tautology

(1) $\sim p \wedge (p \vee q) \rightarrow q$

(2) $\sim p \vee (p \vee q) \rightarrow q$

(3) $\sim p \vee (p \wedge q) \rightarrow q$

(4) None of these

Ans. (1)

Sol. (i) $\sim p \wedge (p \vee q) \rightarrow q$

$(\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$C \vee (\sim p \wedge p) \rightarrow q$

$(\sim p \wedge q) \rightarrow q$

$\sim (\sim p \wedge q) \vee q$

$= (p \vee \sim q) \vee q = p \vee t = t$

(ii) $\sim p \vee (p \vee q) \rightarrow q$

$(\sim p \vee p) \vee q \rightarrow q$

$t \vee q \rightarrow q$

$t \rightarrow q$

(iii) $\sim p \vee (p \wedge q) \rightarrow q$

$\Rightarrow (\sim p \vee p) \wedge (\sim p \vee q) \rightarrow q$

$t \wedge (\sim p \vee q) \rightarrow q$

$\Rightarrow \sim p \vee q \rightarrow q$

$\sim (\sim p \vee q) \vee q$

$(p \wedge \sim q) \vee q$

$(q \vee p) \wedge (q \vee \sim q) = q \vee p$