JEE MAINS 2020 4th SEPT SHIFT 1

Physics

Question 1. Dimensional formula of thermal conductivity will be:

(1) $M^1L^1T^{-3}\theta^{-1}$

(2) $M^0L^1T^{-1}\theta^{-1}$

(3) $M^1L^0T^{-1}\theta^{-1}$

(4) $M^1L^1T^0\theta^{-1}$

Ans. (1)

Sol. $k = \dfrac{(Q/t)\Delta x}{A\Delta T}$

$= \dfrac{M^1L^2T^{-2}(L)}{L^2\theta(T)}$

$= M^1L^1T^{-3}\theta^{-1}$

Question 2. Two disc of radius R and $\dfrac{R}{2}$ are made with same material with same thickness. Disc of radius R rotates with speed of ω and disc of radius $\dfrac{R}{2}$ is at rest. Now both disc are placed coaxially. Find percentage loss of kinetic energy when they rotates with same angular velocity.

(1) 10

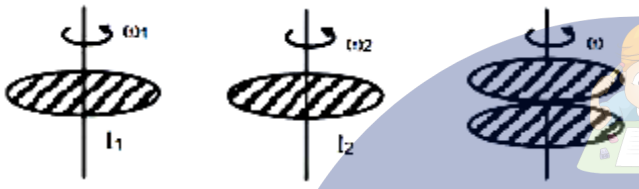
(2) 20

(3) 30

(4) 40

Ans. (2)

Sol.



Angular momentum conservation

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\times\omega_f$$

$$\frac{MR^2}{2}\times\omega + 0 = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f$$

$$\omega_f = \frac{4}{5}\omega$$

Final K.E.

$$K_f = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\frac{16}{25}\omega^2$$

$$K_f = \frac{MR^2\omega^2}{5}$$

$$K_i = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \frac{MR^2\omega^2}{4}$$

Percentage loss in kinetic energy

$$\% \text{ loss} = \frac{\frac{MR^2\omega^2}{4} - \frac{MR^2\omega^2}{5}}{\frac{MR^2\omega^2}{4}} \times 100 = 20\%$$

Question 3. For Lyman series $\lambda_{max} - \lambda_{min} = 340\text{Å}$, Find the same for paschen series?

(1) 11802 Å

(2) 13802 Å

(3) 12502 Å

(4) 10000 Å

Ans. (1)

Sol. Lyman; $\frac{1}{\lambda_{min}} = R(1) = R; n = \infty \text{ to } 1$

$$\frac{1}{\lambda_{max}} = R\left\{1 - \frac{1}{4}\right\} = \frac{3R}{4}; n = 2 \text{ to } 1$$

$$\Rightarrow \lambda_{max} - \lambda_{min} = \frac{4}{3R} - \frac{1}{R}$$

$$340 = \frac{1}{3R} \quad \ldots (1)$$

Paschen: $Y_{\lambda'_{min}} = R\left(\frac{1}{9}\right)$ and $Y_{\lambda'_{max}} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{16 \times 9}$

$$\lambda'_{max} - \lambda'_{min} = \frac{16\times 9}{7R} - \frac{9}{R} = \frac{81}{7R} \ldots\ldots (ii)$$

$$\frac{(ii)}{(i)} = \frac{x}{340} = \frac{81}{7} \quad \Rightarrow x = 11,802.8$$

Question 4. A body of mass $\frac{m}{2}$ moving with velocity v0 collides elastically with another mass of $\frac{m}{3}$. Find % change in KE of first body?

(1) 32%

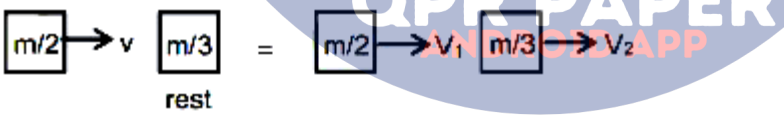
(2) 96%

(3) 34%

(4) 80%

Ans. (2)

Sol.



$$v_1 = \frac{2(m/3)0}{\left(\frac{m}{2}+\frac{m}{3}\right)} + \frac{\left(\frac{m}{2}-\frac{m}{3}\right)v}{\left(\frac{m}{2}+\frac{m}{3}\right)} = \frac{v}{5}$$

for body of m/2

$$K_i = \frac{1}{2}\left(\frac{M}{2}\right)V^2 = \frac{1}{4}mv^2$$

$$K_f = \frac{1}{2}\left(\frac{m}{2}\right)\left(\frac{v}{5}\right)^2 = \frac{1}{100}mv^2$$

$$\% \text{ Loss} = \frac{k_i - k_f}{k_i} \times 100 = \frac{\frac{mv^2}{4} - \frac{mv^2}{100}}{\frac{mv^2}{4}} \times 100 = 96\%$$

Question 5. A body of mass m moving with velocity 'v' collides with shown masses respectively. Find loss in KE after the last collision. Consider all collision completely in elastically?

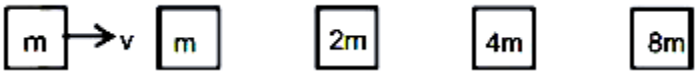
(1) 85.5

(2) 90.2

(3) 93.75

(4) 88.5

Ans. (3)

Sol. 

inelastic collision

$$v^{1} = \frac{v}{16}$$

$$\Delta K \text{ loss} = \frac{1}{2}mv^{2} - \frac{1}{2}(16M)\left(\frac{v}{16}\right)^{2}$$

$$= \frac{1}{2}mv^{2} - \frac{1}{2}M\frac{v^{2}}{16}$$

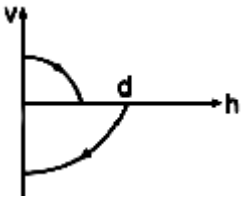
$$= \frac{1}{2}mv^{2}\left(\frac{15}{16}\right)$$

$$\% \ \Delta K \text{ loss} = \frac{\frac{1}{2}mv^{2}\left(\frac{15}{16}\right)}{\frac{1}{2}MV^{2}} \times 100$$

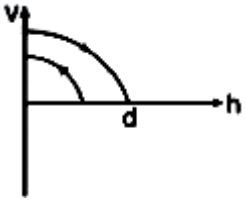
$$= \frac{15}{16} \times 100 = 93.75\%$$

Question 6. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

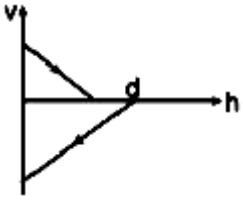
(1)



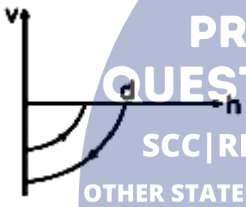
(2)



(3)



(4)



Ans. (1)

Sol. (A)

(i) For uniformly accelerated/deaccelerated motion

$$v^2 = u^2 \pm 2gh$$

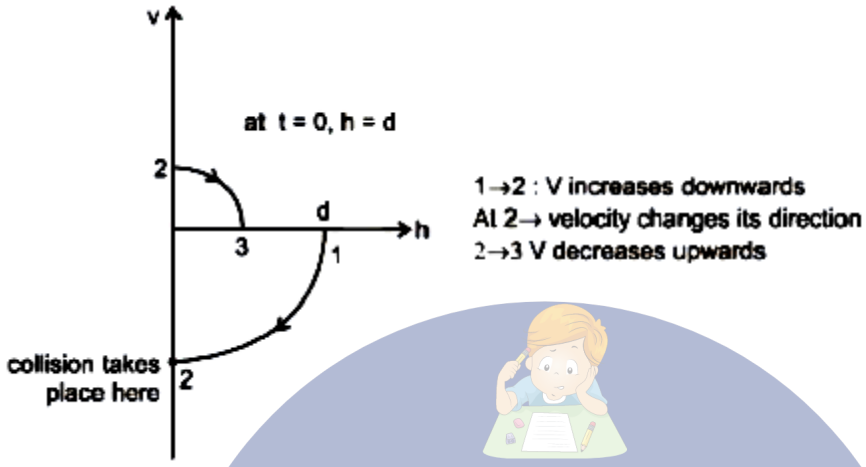
i.e. v-h graph will be a parabola (because equation is quadratic).

(ii) Initially velocity is downwards (-ve) and then after collision it reverses its direction with lesser magnitude i.e. velocity is upwards (+ve). Graph (A) satisfies both these conditions.

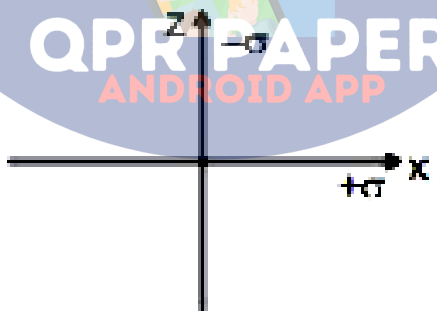
Therefore, correct answer is (A)

Note that time t = 0 corresponds to the point on the graph where h = d

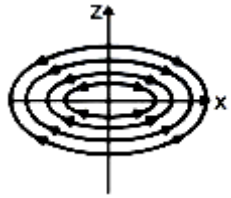
Next time collision takes place at 3.



Question 7. Two infinitely large charged planes having uniform surface charge density $+\sigma$ and $-\sigma$ are placed along x-y plane and yz plane respectively as shown in the figure. Then the nature of electric lines of forces in x-z plane is given by:



(1)



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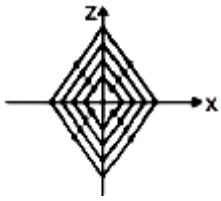

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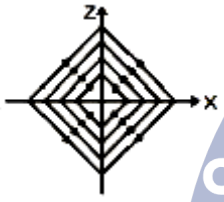

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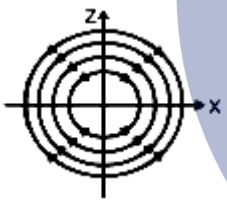
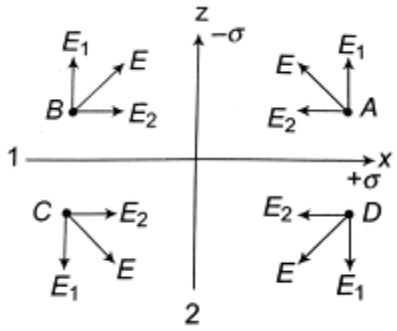


Ans. (3)

Sol. The electric field intensity due to each uniformly charged infinite plane is uniform. The electric field intensity at points A, B, C and D due to plane 1, plane 2 and both planes are given by E1, E2 and E as shown in figure 1. Hence the electric lines of forces are as given in figure 3.



Question 8. Gravitational field intensity is given by $E = \frac{Ax}{\left(A^2 + x^2\right)^{3/2}}$, then find out potential at x.

(Assume potential at infinity = 0)

(1) $-\frac{2A}{\sqrt{A^2 + x^2}}$

(2) $-\frac{A}{\sqrt{A^2 + x^2}}$

(3) $-\frac{A}{3\sqrt{A^2 + x^2}}$

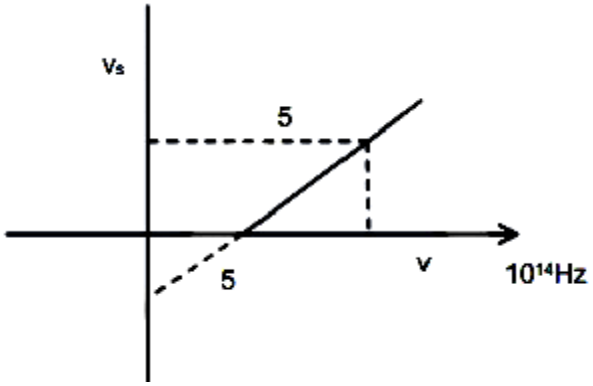
(4) $-\frac{3A}{\sqrt{A^2 + x^2}}$

Ans. (2)

Sol. $V_x = -\int_{\infty}^{x} \frac{Ax}{\left(A^2 + x^2\right)^{3/2}}(-dx)$

$$V_x = -\frac{A}{\sqrt{A^2 + x^2}}$$

Question 9. Graph between stopping potential and frequency of light as shown in figure.



(1) 4.01

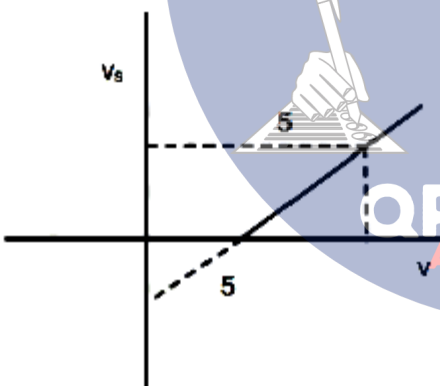
(2) 2.01

(3) 5.01

(4) 2.04

Ans. (2)

Sol.



Threshold Energy = hv

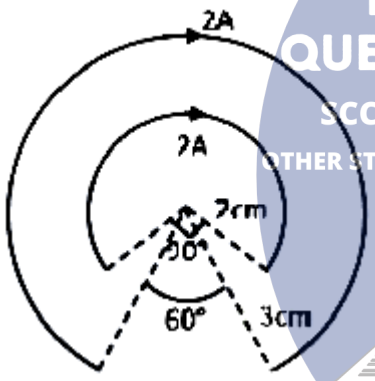
= 6.6 × 10-34 × 5 × 1014 J

$$\text{work function} = \frac{6.6\times5\times10^{-20}}{1.6\times10^{-19}}\text{eV}$$

$$= \frac{6.6 \times 5}{1.6} \times 10^{-1}\,\text{eV}$$

$$= \frac{3.3}{1.6} = 2.01\,\text{eV}$$

Question 10. Two concentric circular current carrying arc of radius R1 = 3cm and R2 = 2cm and direction of current in both arc are shown in figure. Find the ratio of magnetic field $\left(\frac{B_1}{B_2}\right)$ at centre produced by both arc.



(1) $\frac{3}{5}$

(2) $\frac{5}{6}$

(3) $\frac{3}{4}$

(4) $\frac{4}{3}$

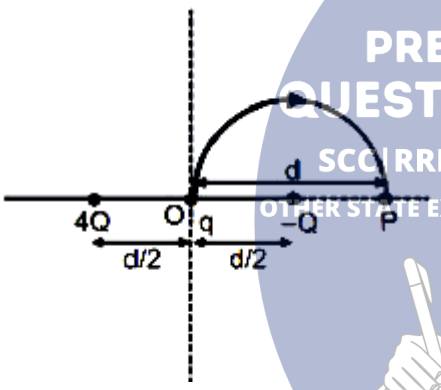
Ans. (1)

Sol. $B_C = \frac{\mu_0 I}{4\pi R}(\theta)\,(\theta \text{ angle substance of centre})$

$$\frac{B_{large}}{B_{small}} = \frac{i_1}{i_2} \times \frac{R_2}{R_1} \frac{(2\pi - \pi/2)}{(2\pi - \pi/3)}$$

$$= \frac{2}{2} \times \frac{2}{3} \times \frac{3\pi}{2} \times \frac{3}{5\pi} = \frac{3}{5}$$

Question 11. Find change in potential energy from origin to point P of charge q moving on the path as shown in figure.



(1) $-\frac{10KQ}{3d}$

(2) $-\frac{13KQ}{3d}$

(3) $-\frac{13KQ}{d}$

(4) $-\frac{16KQ}{3d}$

Ans. (4)

Sol. Potential at O,

$$\Rightarrow V_0 = \frac{K4Q}{\frac{d}{2}} + \frac{K(-Q)}{\frac{d}{2}} = \frac{6KQ}{d}$$

Potential at P,

$$\Rightarrow V_P = \frac{K4Q}{\frac{3d}{2}} + \frac{K(-Q)}{\frac{d}{2}} = \frac{2KQ}{3d}$$

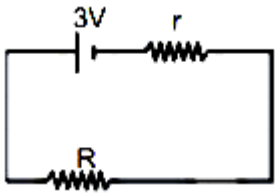
Change in potential energy of a charge

$$q = q\Delta V = q(V_f - V_i)$$

$$= q(V_P - V_0)$$

$$q = \left(\frac{2KQ}{3d} - \frac{6KQ}{d}\right) = -\frac{16KQ}{3d}$$

Question 12. Terminal voltage of cell (emf = 3V & internal resistance = r) is equal to 2.5V and heat loss in R is given by 0.5 watt, then find power loss in internal resistance.



(1) 0.3

(2) 0.5

(3) 0.1

(4) 1

Ans. (3)

Sol. E = 3V

VR = 2.5V

By KVL

Vr + VR = E

Vr + 2.5 = 3

Vr = 0.5

$$\frac{V_R}{V_r} = \frac{IR}{Ir} = \frac{2.5}{0.5} = 5 \quad \ldots\ldots (1)$$

$$\frac{R}{r} = 5$$

$$\frac{P_R}{P_r} = \frac{I^2R}{I^2r} = \frac{R}{r}$$

$$\frac{P_R}{P_r} = 5$$

$$P_r = \frac{P_R}{5} = \frac{0.5}{5} = 0.1 \text{ watt}$$

Question 13. Correct order of wavelength will be:

(1) Radio waves > microwaves > visible rays > X-rays

(2) Microwaves > Radio waves > Visible rays > X-rays

(3) X-rays > Radio waves > Microwaves > Visible rays

(4) X-rays > Radio waves > Visible rays > Microwaves

Ans. (1)

Sol. Theory based

Question 14. A particle at origin (0, 0) moving with initial velocity u = 5 m/s $\hat{j}$ and acceleration$10\hat{i}+4\hat{j}$. After t time it reaches at position (20, y) then find t and y:

(1) t = 2, y = 18

(2) t = 4, y = 16

(3) t = 6, y = 12

(4) t = 8, y = 10

Ans. (1)

Sol. Equation (1)

$$S_x = \frac{1}{2}a_x t^2$$

$$20 = \frac{1}{2}\times 10 \times t^2$$

$$t = 2$$

Equation (2)

$$S_y = u_y t + \frac{1}{2}a_y t^2$$

$$y = 5(2) + \frac{1}{2}(4)(2)^2$$

$$y = 18$$

Question 15. Distance between trough and crest of a waves is 1.5 m while distance between two through is 5m. Which of the following wavelength is possible.

(1) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots\ldots$

(2) 1, 2, 3, ...

(3) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \ldots\ldots$

(4) 1, 3, 5,....

Ans. (3)

Sol. Trough to crest distance

$$1.5 = (2n_1 + 1)\frac{\lambda}{2} \quad \ldots (1)$$

Trough to trough distance

$$5 = (n_2\lambda) \quad \ldots (2)$$

from (1) and (2)

$$\frac{1.5}{5} = \frac{2n_1 + 1}{2(n_2)}$$

$3n2 = 10n1 + 5$

n1 and n2 are integer

$(1)\ n_1 = 1, n_2 = 5, \lambda = 1$

$(2)\ n_1 = 4, n_2 = 15, \lambda = \frac{1}{3}$

$(3)\ n_1 = 7, n_2 = 25, \lambda = \frac{1}{5}$

Question 16. Intensity of plane polarized light is 3.3 W/m. Area of a plane 3×10^{-4} m^2 and polarizer rotates with 10π rad/sec. Energy transmitted in 1 complete cycle:

(1) 4.95×10^{-4}

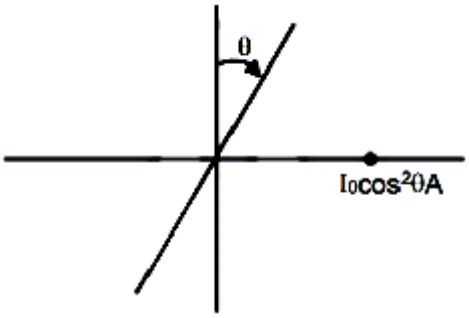
(2) 3.95×10^{-4}

(3) 2.95×10^{-4}

(4) 6.95×10^{-4}

Ans. (1)

Sol.

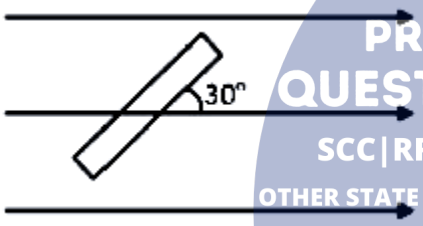


Average energy $= I_0 A < \cos^2 \theta >$

$$= \frac{3.3 \times 3 \times 10^{-4}}{2}$$

$$= \frac{9.9}{2} \times 10^{-4} = 4.95 \times 10^{-4}$$

Question 17. A bar magnet experienced torque 0.018 N-m when placed in uniform magnetic field, B = 0.06 T and makes 30° angle with the magnetic field as shown in figure. Find out work done by external force if magnet rotates from minimum potential energy to maximum potential energy.



(1) 0.036 J

(2) 0.018 J

(3) 0.072 J

(4) 0.36 J

Ans. (3)

Sol. $\tau = MB\sin\theta = 0.018$

$$M = \frac{0.018}{B\sin\theta} = \frac{0.018}{0.06 \times 0.5} = 0.6A - m^2$$

$$\omega = \Delta U = U_f - U_i$$

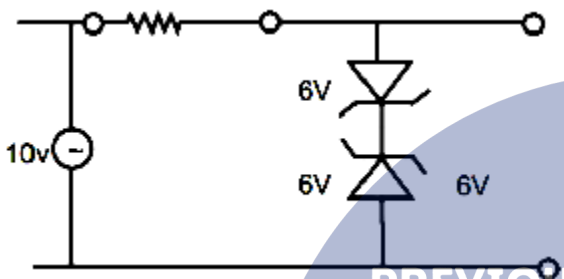
$$= -MB\cos 180° - \left(-MB\cos 0°\right)$$

$= 2MB$

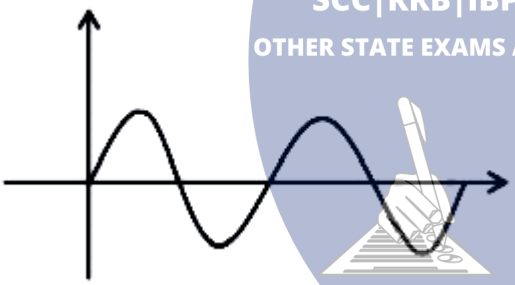
$= 2 \times 0.6 \times 0.06$

$= 0.072$ J

Question 18. Correct graph of voltage across zener diode will be



(1)



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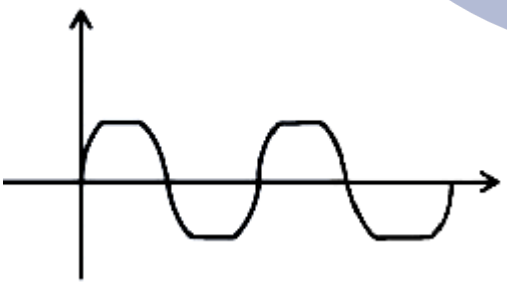

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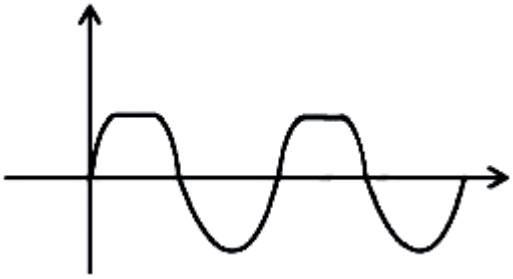

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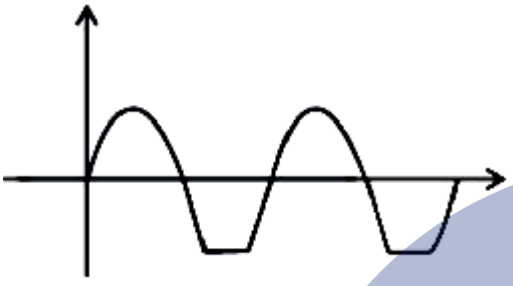
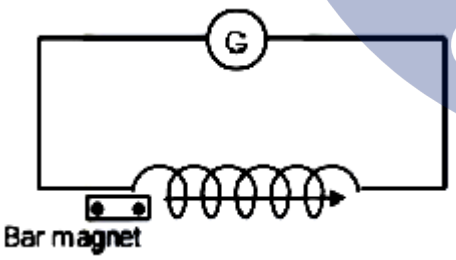


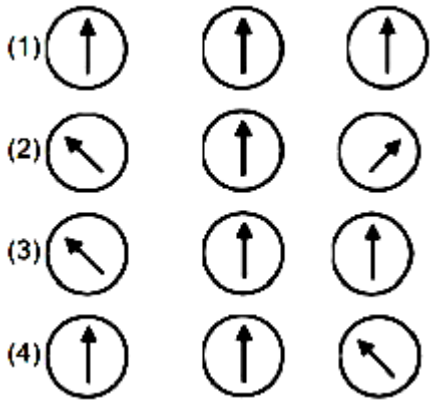
Ans. (2)

Sol. Voltage across zener diode remains constant and as there are two zener diodes, so voltage remains constant for both direction of AC current.

Question 19.



A bar magnet moves with constant velocity as shown in figure through a coil. Which of the following option is correctly represent the deflection of needle in Galvanometer.



Ans. (2)

Sol. When magnet enter or exit the coil current generated due to flux change. When magnet is in coil there is no change in flux, so galvanometer does not show deflection.

Question 20. In compound microscope final image formed at 25 cm from eyepiece lens. Length of tube is 20cm. Given that $f_0 = 1$ cm, m = 100. Find focal length of eyepiece lens

Ans. (06.25)

Sol. $M = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$

$$M = \frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$$

$$100 = \frac{20}{(1)}\left(1+\frac{25}{f_e}\right)$$

$$5 = 1+\frac{25}{f_e}$$

$$4 = \frac{25}{f_e}$$

$$f_e = \frac{25}{4} = 6.25 \text{ cm}$$

Question 21. 0.1 mole of a gas at 200 K is mixed with 0.05 mole of same gas at 400 K. If final temperature is equal to $10T_0$, then find the value of T_0.

Ans. (26.66)

Sol. $(0.1)(200) + (0.05)(400) = (0.15)\,T$

$$T = \frac{20 + 20}{0.15} = \frac{800}{3} = 266.67$$

$$10T_0 = 266.67$$

$$T_0 = 26.66$$

Chemistry

Question 22. Elements with atomic number 101 and 104 belongs respectively, to:

(1) Actinoids and group 4

(2) Actinoids and group 6

(3) group 11 and group 6

(4) group 4 and group 6

Answer: (1)

Solution:

89Ac → 103Lr

Belongs to actinoids series and they all belongs to 3rd group. So atomic no 101 element is actinoids and atomic number 104 elements belongs to 4th group.

Question 23. Find total possible number of isomers for the complex $[Pt(en)(NO_2)_2]$.

(1) 1

(2) 3

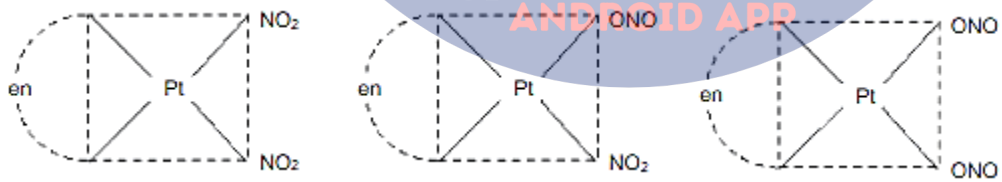
(3) 2

(4) 4

Answer: (2)

Solution:

$[Pt(en)(NO_2)_2]$



So, total possible isomers are 3.

Question 24. Match List-I and List-II and select the correct using the code given below the lists.

	List-I		List-II

(a)	Foam	(i)	Smoke
(b)	Gel	(ii)	Gem Stone
(c)	Aerosol	(iii)	Froath
(d)	Emulsions	(iv)	Jelly
		(v)	Rubber
		(vi)	Milk

(1) (a) – (iii), (b) – (iv), (c) – (i), (d) – (vi)

(2) (a) – (iii), (b) – (iv), (c) – (v), (d) – (vi)

(3) (a) – (ii), (b) – (iv), (c) – (i), (d) – (iii)

(4) (a) – (iii), (b) – (ii), (c) – (i), (d) – (vi)

Answer: (1)

Solution:

Dispersed Phase	Dispersion medium	Type of colloids	Examples
Solid	Solid	Solid sol	Some coloured glasses and gem stones
Solid	Liquid	Sol	Paints, cell fluids
Solid	Gas	Aerosol	Smoke, dust
Liquid	Solid	Gel	Cheese, butter, jellies
Liquid	Liquid	Emulsions	Milk, hair cream
Liquid	Gas	Aerosol	Fog, Mist, cloud, insecticide sprays
Gas	Solid	Solid sol	Pumice stone, foam rubber

Gas	Liquid	Foam	Froth, whipped cream, soap lather

Question 25. Correct order of ionic radii in: O_2, F, Mg^{2+}, Na^+.

(1) $Mg^{2+} > Na^+ > F > O_2$

(2) $Na^+ > O_2 > F > Mg^{2+}$

(3) $O_2 > F > Na^+ > Mg^{2+}$

(4) $O_2 > Na^+ > F > Mg^{2+}$

Answer: (3)

Solution:

	O_2	F	Na^+	Mg^{2+}
Z	8	9	11	12
No. of e	10	10	10	10

In isoelectronic species greater is Z smaller is radius so order is $O_2 > F > Na^+ > Mg^{2+}$

Question 26. Which oxides are formed on heating Li, Na, K in excess of O_2

(1) Li_2O, Na_2O_2, KO_2

(2) Li_2O, Na_2O_2, K_2O

(3) Li_2O, Na_2O, K_2O

(4) Li_2O_2, Na_2O, K_2O_2

Answer: (1)

Solution:

On heating in excess air Li form oxide sodium form peroxide while K, Rb, Cs form superoxide.

Question 27. Which of the following have same magnetic moment (spin only)

(1) $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{2+}$

(2) $[Cr(H_2O)_6]^{2+}$, $[CoCl_6]^{2+}$

(3) $[Cr(H_2O)_6]^{2+}$, $[CoCl_4]^{2+}$

(4) $[Co(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{2+}$

Answer: (1)

Solution:

No. of unpaired e^-

$[Cr(H_2O)_6]^{2+} \Rightarrow Cr^{2+} \Rightarrow 3d_4 \Rightarrow$ t2 g1,1,1, eg1,0
4

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d_6 \Rightarrow$ t2 g2,1,1, eg1,1
4

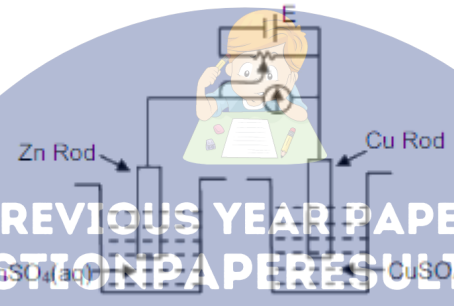
$[CoCl_6]^{2+} \Rightarrow Co^{4+} \Rightarrow 3d_5 \Rightarrow$ t2 g1,1,1, eg1,1
5

$[CoCl_4]^{2+} \Rightarrow Co^{2+} \Rightarrow 3d_7 \Rightarrow$ t2 g2,2,1, eg1,1
3

$[Co(H_2O)_6]^{2+} \Rightarrow Co^{2+} \Rightarrow 3d_7 \Rightarrow t2\ g2,2,1,\ eg1,1$
3

So $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same magnetic moment spin only

Question 28. For the given cell arrangement identify incorrect statement.



[Given $E^0_{Cu^{2+}|Cu} = 0.34V$ & $E^0_{Zn^{2+}|Zn} = 0.76V$]

(1) At E = 1.1 V no electron will flow

(2) At E > 1.1 V no electron will flow form Cu to Zn

(3) At E < 1.1 V Zn will dissolve and Cu will deposit

(4) At E < 1.1 V Zn will deposit and Cu will dissolve

Answer: (4)

Solution:

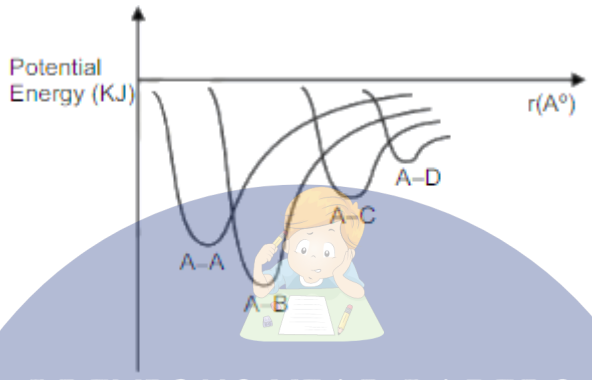
$$E^0_{cell} = E^0_{Cu^{2+}|Cu} - E^0_{Zn^{2+}|Zn} = 1.1V$$

So if E = 1.1 V no electron will flow

At E > 1.1 V cell act as electrolytic cell and electron will flow from Cu to Zn.

At E < 1.1V cell act as electrochemical cell so Zn dissolve and Cu deposit.

Question 29. Using following potential energy graph identify correct option.



(1) D has highest electronegativity

(2) A – B has most stiff bond.

(3) A – D has minimum bond length

(4) Bond length of A – B is greater than A – C bond

Answer: (2)

Solution:

Bond enthalpy of AB bond is highest so A – B bond is more strong and B is highest electronegativity atom. Order of bond length $\Rightarrow$ A – A < A – B < A – C < A – D

Question 30. When lead (II) nitrate is heated a brown gas 'B' is evolved, this gas 'B' on cooling give a colourless solid 'C'. Gas 'B' on reaction with NO forms a blue solid 'D'. Find oxidation state of 'N' in D;

(1) +2

(2) +3

(3) +4

(4) +5

Answer: (2)

Solution:

$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2(g)$(brown gas)

$2NO_2(g) \xrightarrow{\text{cooling}} N_2O_4$

'B' 'C'

$NO_2(g) + NO(g) \rightarrow \overset{+3}{N_2O_3}$

'D'

Question 31. Select the correct set of the following statements:

(a) In the metallurgy of iron, lime stone is converted to CaO.

(b) Silver is extracted as anionic complex

(c) Nickel is purified by mond's process.

(d) Zr & Ti are purified by Van Arkel process.

(1) a, b, d

(2) a, c

(3) b, c, d

(4) a, b, c, d

Answer: (4)

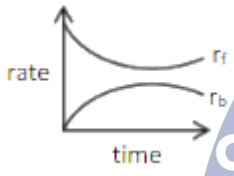
Solution:

All statements are correct

Question 32. At equilibrium for a reaction A $\rightleftharpoons$ B.

Correct representation is {rf = Rate of forward reaction, rb = Rate of backward reaction}

(1)



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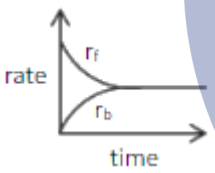
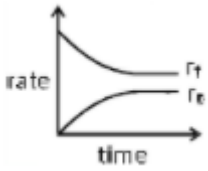


(3)



(4)

Answer: (2)

Solution:

At equilibrium, rate of forward reaction = Rate of backward reaction.

Question 33. In which region lines of Balmer series lie

(1) Visible

(2) Infrared

(3) Ultra violet

(4) Radio wave

Answer: (1)

Solution:

In hydrogen spectrum maximum lines of Blamer series lies in visible region.

Question 34. For 1 mole of ideal gas which of the following statements must be true.

(a) U and H depends only on temperature

(b) Compressibility factor (Z) cannot be 1

(c) Cp – Cv = R

(d) $\Delta U = CvdT$ for all processes

(1) a, c, d

(2) b, c, d

(3) c, d

(4) a, c

Answer: (1)

Solution:

(a) For ideal gas U and H are function of Temperature U

$= \frac{f}{2}$ nRT

(c) Cp – Cv = R

(d) $\Delta U = CvdT$ for all processes

Question 35. 20 mL of 0.2 gram H_2O_2 (impure) reacts completely with 0.316 gram $KMnO_4$. Find percentage purity of H_2O_2. [Given Molecular mass H_2O_2 = 34 R & $KMnO_4$ = 158]

Answer: 85.00

Solution:

Let mass of pure H_2O_2 is x gram

+7 -1 +2 0

$MnO_4 + H_2O_2 \rightarrow Mn^{2+} + O_2$

vf = 5 vf =2

Equation of H2O2 = Eq. of MnO4

$$\left[\frac{x}{34}\right]2 = \left[\frac{0.316}{158}\right]5$$

$x = 0.17$

So, % purity of H_2O_2 solution $= \frac{0.17}{0.2} \times 100 = 85\%$

Question 36. A reaction 75% complete in 90 min, find the time (in min.) taken to complete 60% of reaction [Given log 2 = 0.3, log 2.5 = 0.4]

Answer: 60.00

Solution:

$$T = \frac{2.303}{K} \log\left[\frac{100}{100 - x\%}\right]$$

$$T_{75\%} = \frac{2.303}{K} \log\left[\frac{100}{25}\right] = 90$$

$$T_{60\%} = \frac{2.303}{K} \log\left[\frac{100}{40}\right]$$

$$\frac{T_{75\%}}{T_{60\%}} = \frac{2\log 2}{\log 2.5}$$

$$\Rightarrow \frac{90}{T_{60\%}} = \frac{2 \times 0.3}{0.4}$$

$$T_{60\%} = \frac{90 \times 4}{6} = 60 \text{ min}$$

Question 37. In the preparation of ammonia, mass of N_2 taken is 2.5 kg and mass of hydrogen taken is 1 kg. Find mass of ammonia formed (in grams)

Answer: 3035.7 gram

Solution:

$$\text{Mole of } N_2 = \frac{2500}{28} = \frac{625}{7} \quad \& \text{ Mole of } H_2 = \frac{1000}{2} = 500$$

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

Initial mole $\frac{625}{7}$ 500 Limiting reagent is N_2

Final mole 0 $500 - \frac{3 \times 625}{7}$ $2 \times \frac{625}{7}$

$$\text{Mass of } NH_3 \text{ formed} = \frac{2 \times 625}{7} \times 17 = 3035.7 \text{ gram}$$

Question 38. Vapour pressure of solution obtained by mixing 1 mole of n-hexane and 3 mole of n-heptane is 550 mm of Hg. On mixing 1 mole n-heptane, vapour pressure of solution increases by 10 mm of Hg. Find the vapour pressure of pure n-heptane (in mm of Hg)

Answer: 600.00

Solution:

Ptotal = P0hexane . Xhexane + P0heptane . Xheptane

$$550 = \left[P^{0}_{\text{hexane}}\right] \times \frac{1}{4} + \left[P^{0}_{\text{heptane}}\right] \times \frac{3}{4} \quad ...(1)$$

After mixing 1 mole n-heptane

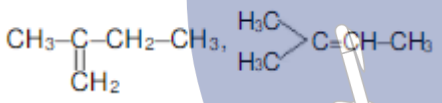
$$560 = \left[P^{0}_{\text{hexane}}\right] \times \frac{1}{5} + \left[P^{0}_{\text{heptane}}\right] \times \frac{4}{5} \quad ...(2)$$

On solving equation (1) and (2)

P0heptane = 600 mm of Hg

Question 39. Neopentyl alcohol on reaction with conc. H_2SO_4 forms two alkenes A & B in the ratio (85:15). Then alkenes A & B respectively are:

(1)



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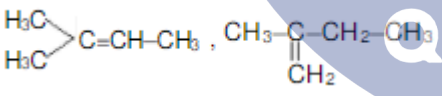

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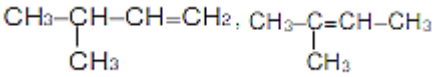
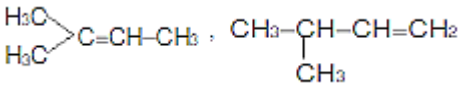
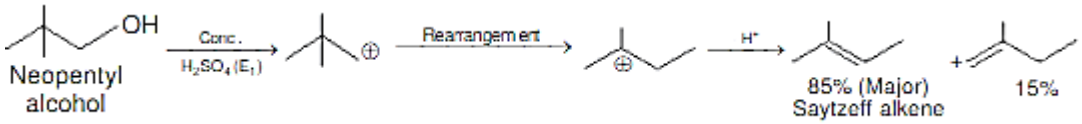


(4)



Answer: (2)

Solution:



Question 40. Aqueous $AgNO_3$ is reacted with the following compounds. Find the order of reactivity.

(1) A > B > C > D

(2) B > A > C > D

(3) B > A > D > C

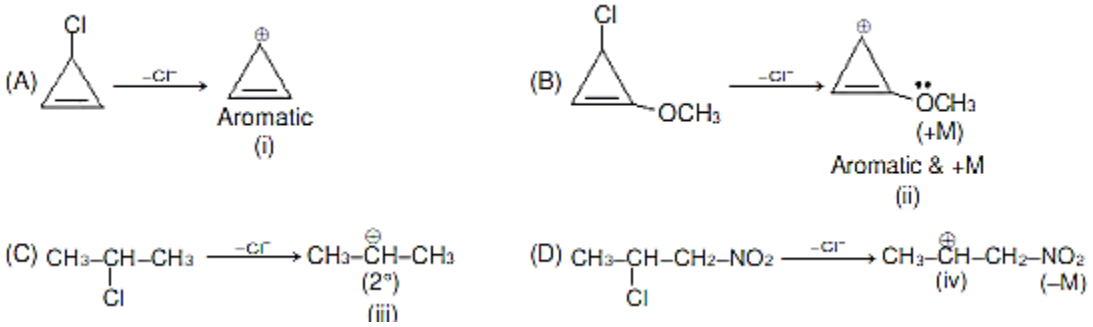
(4) A > C > B > D

Answer: (2)

Solution:

Given reaction is SN_1 reaction. In SN_1 reaction

Rate of reaction $\propto$ Stability of C^+



Stability of C+ : ii > i > iii > iv

Reactivity order : B > A > C > D

Question 41. Which of the following is the correct statement for maltose?

(1) It has one ketal and one hemiketal group

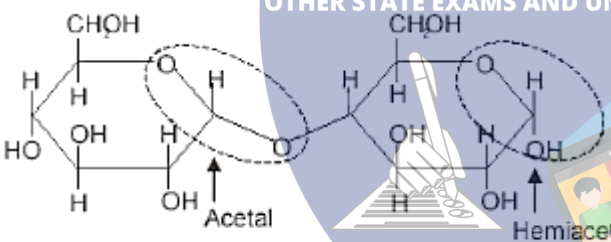
(2) It has one acetal and one hemiacetal group

(3) It has two acetal group

(4) It has one ketal and one hemiketal group

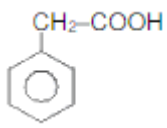
Answer: (2)

Solution:

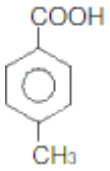


Question 42. A compound (X) on reaction with $Br_2/FeBr_3$ in CCl_4 gives only a single isomer $C_8H_7O_2Br$. (X) on heating with soda lime give Toluene. Find the structure of X.

(1)



(2)



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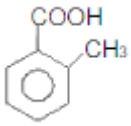

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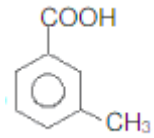
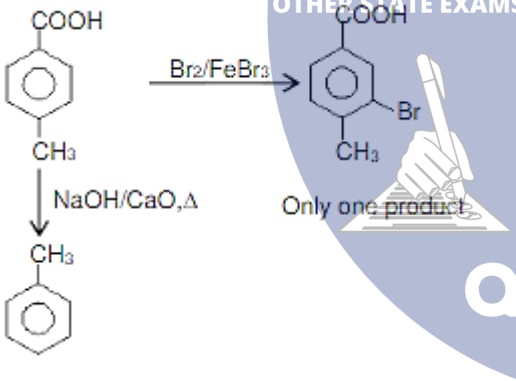


Answer: (2)

Solution:



Question 43. Which of the following reacts with alc. KOH & $CHCl_3$

(1) Adenine & Proline

(2) Adenine & Thiamine

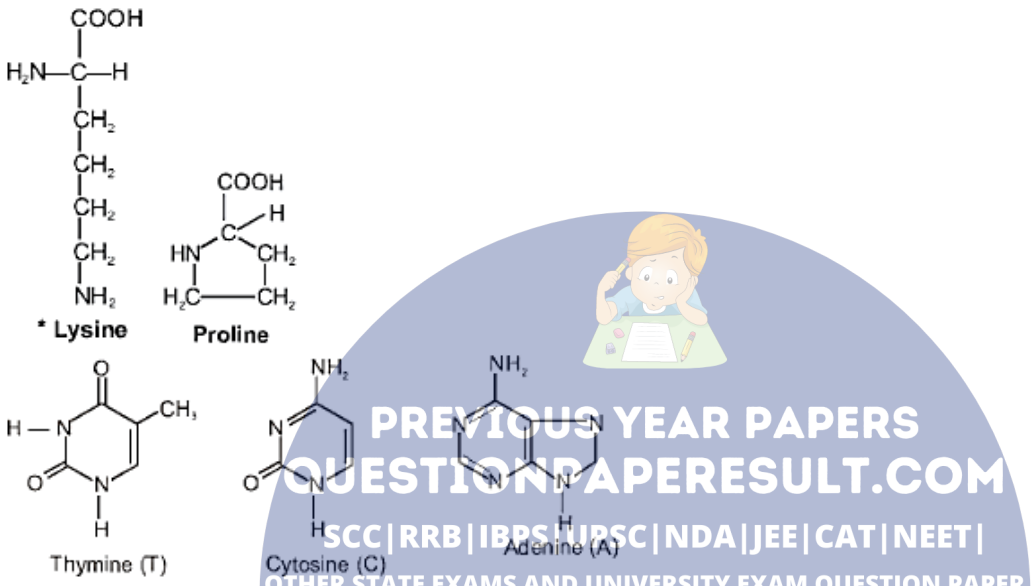
(3) Adenine & Lysine

(4) Cytosine & Proline

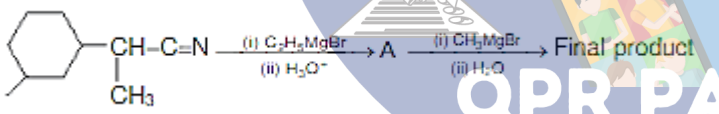
Answer: (3)

Solution:

Compounds having 1o amine give carbylamines with $CHCl_3$ & alc. KOH

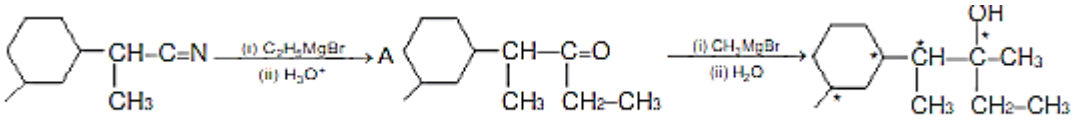


Question 44. Find the number of chiral centres in the final product:



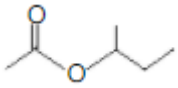
Answer: 4.00

Solution:

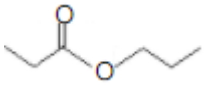


Question 45. $P(C_6H_{12}O_2)$ undergoes acidic hydrolysis to give Q(an alcohol). R gives Lucas test immediately. Then P can be

(1)



(2)



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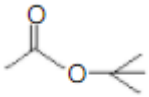

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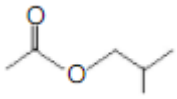
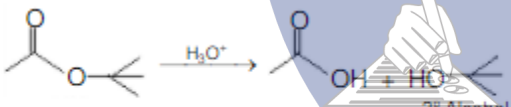


Answer: (3)

Solution:



Mathematics

Question 46. In a group, 63% people read newspaper A while 76% people read newspaper B. If x% people read both A and B then x may be

(1) 37%

(2) 68%

(3) 29%

(4) 55%

Ans. (4)

Sol. $n(A) = 63\%$

$n(B) = 76\%$

Let $n(A \cap B) = x\%$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$

$63 + 76 - x \leq 100$

$x \geq 39$

But $n(A \cap B) \leq n(A) \therefore 39 \leq x \leq 63.$

Question 47. If $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$, then find the value of

$f(3) - f(1)$

(1) $\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(2) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3) $\frac{\pi}{12} + \frac{1}{3} - \frac{\sqrt{3}}{4}$

(4) $\frac{\pi}{12} + \frac{1}{4} - \frac{\sqrt{3}}{4}$

Ans. (2)

Sol. $f(x)=\int\frac{\sqrt{x}}{(1+x)^2}dx$

Let, $x=\tan^2\theta$

$dx=2\tan\theta\sec^2\theta d\theta$

$f(x)=\int\frac{\tan\theta}{(1+\tan^2\theta)^2}.2\tan\theta\sec^2\theta d\theta$

$f(x)=\int\frac{\tan\theta}{\sec^4\theta}.2\tan\theta\sec^2\theta d\theta$

$f(x)=\int 2\tan^2\theta\cos^2\theta d\theta$

$f(x)=\int 2\sin^2\theta d\theta$

$f(x)=\int(1-\cos 2\theta)d\theta$

$f(x)=\theta-\frac{\sin 2\theta}{2}+C=\theta-\frac{\tan\theta}{1+\tan^2\theta}+C$

$f(x)=\tan^{-1}\sqrt{x}-\frac{\sqrt{x}}{1+x}+C$

Now $f(3)-f(1)=\tan^{-1}\sqrt{3}-\frac{\sqrt{3}}{1+3}-\tan^{-1}\sqrt{1}+\frac{1}{1+1}$

$=\frac{\pi}{3}-\frac{\pi}{4}+\frac{1}{2}-\frac{\sqrt{3}}{4}=\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Question 48. Let ΔABC is a right angled triangle, right angled at A such that A(1, 2), C(3, 1) and area of $\Delta ABC = 5\sqrt{5}$ then abscissa of B can be

(1) $1+5\sqrt{2}$

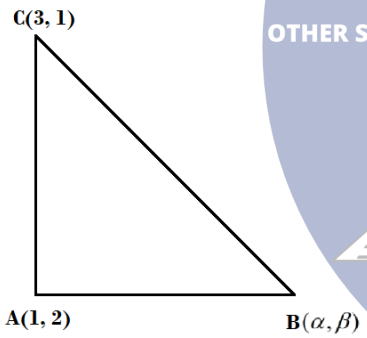
(2) $1+2\sqrt{5}$

(3) $1-5\sqrt{2}$

(4) $3+2\sqrt{5}$

Ans. (2)

Sol.



$$m_{AB} = \frac{\beta-2}{\alpha-1}$$

$$m_{AC} = \frac{1-2}{3-1} = -\frac{1}{2}$$

$AB \perp AC \qquad \therefore \left(\frac{\beta-2}{\alpha-1}\right)\left(-\frac{1}{2}\right) = -1$

$\beta = 2\alpha - 2 + 2 \quad \Rightarrow \beta = 2\alpha$

Now area of $\Delta ABC = 5\sqrt{5} = \frac{1}{2} AB \times AC$

$$\Rightarrow \frac{1}{2}\sqrt{(3-1)^2+(1-2)^2}.\sqrt{(\alpha-1)^2+(\beta-2)^2} = 5\sqrt{5}$$

$$\Rightarrow \sqrt{(\alpha-1)^2+(2\alpha-2)^2} = 10$$

$$\Rightarrow \sqrt{(\alpha-1)^2}\sqrt{5} = 10 \quad \Rightarrow |\alpha-1| = 2\sqrt{5} \quad \Rightarrow \alpha = 1 \pm 2\sqrt{5}$$

Question 49. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0,4]$, then $\int_0^3 (g(x) - f(x))\,dx =$

(1) 1

(2) 2

(3) 3

(4) 4

Ans. (1)

Sol. $f(x) = |x-2| = \begin{cases} 2-x & x < 2 \\ x-2 & x \geq 2 \end{cases}$

$$g(x) = f(f(x)) = \begin{cases} 2-f(x) & f(x) < 2 \\ f(x)-2 & f(x) \geq 2 \end{cases}$$

$$= \begin{cases} 2-(2-x) & 2-x<2, \quad x<2 \\ (2-x)-2 & 2-x\geq 2, \quad x<2 \\ 2-(x-2) & x-2<2, \quad x\geq 2 \\ (x-2)-2 & x-2\geq 2 \quad x\geq 2 \end{cases}$$

$$= \begin{cases} x & 0<x<2 \\ -x & x\leq 0 \\ 4-x & 2\leq x<4 \\ x-4 & x\geq 4 \end{cases}$$

Now,

$$\int_0^3 (g(x)-f(x))dx = \int_0^2 x\,dx + \int_2^3 (4-x)dx$$

$$-\left[\int_0^2 (2-x)dx + \int_2^3 (x-2)dx\right]$$

$$= \int_0^2 (2x-2)dx + \int_2^3 (6-2x)dx$$

$$= \left[x^2-2x\right]_0^2 + \left[6x-x^2\right]_2^3$$

$$= (4-4-0)+(18-9-12+4) = 1$$

Question 50. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

(1) ${}^{51}C_7 - {}^{30}C_6$

(2) ${}^{51}C_6 - {}^{30}C_6$

(3) ${}^{51}C_7 - {}^{30}C_7$

(4) ${}^{51}C_6 - {}^{30}C_7$

Ans. (3)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

$= 50C6 + 49C6 + 48C6 + \ldots\ldots + 30C6 = -30C7 + 30C7 + 30C6 + 31C6 + 32C6 + \ldots. + 50C6$

$= -30C7 + 31C7 + 31C6 + 32C6 + \ldots\ldots + 50C6$

$= -30C7 + 32C7 + 32C6 + \ldots. + 50C6$

$= -30C7 + 51C7.$

Question 51. Let $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x)$ is a differential equation. If $f(\pi) = \pi$, then $f'\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) =$.

(1) $\frac{\pi}{2} + 2$

(2) $\frac{\pi}{2} - 2$

(3) $\frac{\pi}{2} + 1$

(4) $\frac{\pi}{2} - 1$

Ans. (1)

Sol. Given $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x)$

$$\Rightarrow \frac{dy}{dx}-\frac{1}{x}y = x\left(x\cos x+\sin x\right)$$

$$\therefore \text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$\therefore \text{ solution is } y.\frac{1}{x} = \int \frac{1}{x}.x\left(x\cos x+\sin x\right)dx + C$$

$$\frac{y}{x} = \int \left(x\cos x+\sin x\right)dx + C$$

$$\frac{y}{x} = x\sin x + C$$

$$y = x^2\sin x + Cx$$

$$\pi = 0 + C\pi \Rightarrow C = 1$$

$$\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$$

$$\frac{d^2y}{dx^2} = -x^2\sin x + 2x\cos x + 2x\cos x + 2\sin x$$

$$= -x^2\sin x + 4x\cos x + 2\sin x$$

$$= f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = \left(-\frac{\pi^2}{4} + 4.0 + 2\right) + \left(\frac{\pi^2}{4}.1 + \frac{\pi}{2}\right)$$

$$= -\frac{\pi^2}{4} + 2 + \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$= \frac{\pi}{2} + 2.$$

Question 52. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be an ellipse such that LR = 10 and its eccentricity is equal to maximum value of quadratic expression $f(t)=\frac{5}{12}+t-t^2$ then $(a^2+b^2)=$

Ans. (126)

Sol. $LR=\frac{2b^2}{a}=10\Rightarrow b2=5a$

$$f(t)=\frac{5}{12}-\left(t^2-t+\frac{1}{4}-\frac{1}{4}\right)=\frac{5}{12}+\frac{1}{4}-\left(t-\frac{1}{2}\right)^2$$

$$=\frac{2}{3}-\left(t-\frac{1}{2}\right)^2$$

$$f(t)_{max}=\frac{2}{3}=e$$

$$5a=a^2\left(1-\frac{4}{9}\right)\Rightarrow 5=\frac{5}{9}a$$

$$\Rightarrow a^2=81 \text{ and } b^2=45$$

Hence, $a^2+b^2 = 126$

Question 53. If α,β are roots of $x^2-3x+p=0$ and γ,δ are roots of $x^2-6x+q=0$ and $\alpha,\beta,\gamma,\delta$ are in increasing geometric progression then value of $\frac{2q+p}{2q-p}$ is equal to

(1) $\frac{7}{9}$

(2) $-\frac{7}{9}$

(3) $\frac{9}{7}$

(4) $-\frac{9}{7}$

Ans. (3)

Sol. $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$\alpha + \beta = 3 \Rightarrow a + ar = 3 \quad \ldots(i)$

$\gamma + \delta = 6 \Rightarrow ar^2 + ar^3 = 6 \quad \ldots(2)$

By (1) and (2) $\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{6}{3} \Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$

Also, $a.ar = p \;\; \& \;\; ar^2.ar^3 = q$

$\therefore \frac{2q+p}{2q-p} = \frac{2(a^2r^5)+a^2r}{2(a^2r^5)-a^2r} = \frac{(2r^4+1)}{(2r^4-1)} = \frac{9}{7}$

Question 54. The mean and variance of 5, 7, 12, 10, 15, 14, a, b are 10 and 13.5 respectively then value of |a - b| =

(1) 5

(2) 6

(3) 7

(4) 8

Ans. (3)

Sol. $\frac{5+7+12+10+15+14+a+b}{8}=10$

$\Rightarrow 63+a+b=80 \Rightarrow a+b=17 \quad ..(1)$

$\sigma^2=\frac{\Sigma x_i^2}{n}-\left(\frac{\Sigma x_i}{n}\right)^2$

$\Rightarrow 13.5=\frac{25+49+144+100+225+196+a^2+b^2}{8}-(10)^2$

$908 = a^2 + b^2 + 739$

$a^2 + b^2 = 169$

$(a + b)^2 - 2ab = 169$

$289 - 169 = 2ab \Rightarrow ab = 60$

$\therefore |a - b|^2 = (a + b)2 - 4ab = 289 - 240 = 49$

$\therefore |a - b| = 7$

Question 55. If $1 + (1 - 22.\ 1) + (1 - 42.\ 3) + (1 - 62.\ 5) +$ …….$+ (1 - 202.\ 19) = \alpha - 220\beta$ then $(\alpha, \beta)$is:

(1) (11, 97)

(2) (11, 103)

(3) (10, 97)

(4) (10, 103)

Ans. (2)

Sol.

$$S = 1 + \sum_{r=1}^{10} 1 - (2r)^2 (2r-1) = 1 + 10 - \sum_{r=1}^{10} (8r^3 - 4r^2)$$

$$= 11 - \left[8\left(\frac{10 \times 11}{2} \right)^2 - 4.\left(\frac{10 \times 11 \times 21}{6} \right) \right]$$

$\alpha - 220\beta = 11 - [2(110)2 - 140 \times 11]$

$\alpha - 220\beta = 11 - 22\,(1100 - 70)$

$\alpha - 220\beta = 11 - 220\,(110 - 7)$

$\therefore \alpha = 11, \beta = 103$

$(\alpha, \beta) = (11, 103)$.

Question 56. For equation $[x]^2 + 2[x + 2] - 7 = 0$, $x \in R$, number of solution of equation is/are

(1) four integer solutions

(2) infinite solutions

(3) No solution

(4) two solutions

Ans. (2)

Sol. $[x]2 + 2[x + 2] - 7 = 0$

$[x]2 + 2([x] + 2] - 7 = 0$

Let $[x] = t$

$t2 + 2t - 3 = 0$

$t = 1, -3$

$[x] = -3, 1$

$x \in [-3,-2) \cup [1,2)$

Hence infinite solution.

Question 57. Integration: $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$ is equal to

(1) $\frac{\sin x + x \cos x}{x \sin x + \cos x} + C$

(2) $\frac{\sin x + x \cos x}{x \sin x - \cos x} + C$

(3) $\frac{x \cos x - \sin x}{\cos x - x \sin x} + C$

(4) $\frac{\sin x - x \cos x}{x \sin x + \cos x} + C$

Ans. (4)

Sol. $\int \frac{x^2\, dx}{(x \sin x + \cos x)^2} = \int \frac{x}{\cos x} \cdot \frac{x \cos x}{(x \sin x + \cos x)^2} dx$

$$= \frac{x}{\cos x}\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx -$$

$$\int\left[\frac{d}{dx}(x\sec x)\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx\right]dx$$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)$$

$$-\int \sec^2 x\,(x\sin x+\cos x)\left(-\frac{1}{x\sin x+\cos x}\right)dx$$

$$\because \frac{d}{dx}(x\sec x) = \sec x + x\sec x\tan x = \sec x\left(1+\frac{x\sin x}{\cos x}\right)$$

$$= \sec^2 x\,(x\sin x+\cos x)$$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)+\int \sec^2 x\,dx$$

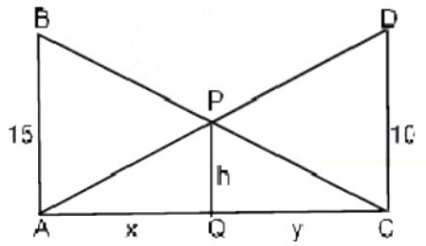
$$= \frac{-x}{\cos x\,(x\sin x+\cos x)}+\frac{\sin x}{\cos x}+C$$

$$= \frac{-x+x\sin^2 x+\sin x\cos x}{\cos x\,(x\sin x+\cos x)}+C = \frac{\sin x - x\cos x}{x\sin x+\cos x}+C$$

Question 58. Two poles AB and CD of height 15m and 10m respectively. A and C are on level ground. Point of intersection of AD and BC is P then height of P is

Ans. (6)

Sol.



$$\Delta AQP \sim \Delta ACD \Rightarrow \frac{x}{h} = \frac{x+y}{10} \quad \ldots. (1)$$

$$\because \Delta CQP \sim \Delta CAB \Rightarrow \frac{y}{h} = \frac{x+y}{15} \quad \ldots. (2)$$

$$(1)+(2) \rightarrow \frac{x+y}{h} = (x+y)\left(\frac{1}{10}+\frac{1}{15}\right) \Rightarrow h = 6$$

Question 59. Consider two statements

$S_1 :\sim p \rightarrow (\sim q \leftrightarrow \sim p)$ is a tautology

$S_2 : (\sim q \wedge p) \rightarrow q$ is a fallacy then

(1) Statement I is true, statement II is false

(2) Statement I is false, statement II is true

(3) Both true

(4) Both false

Ans. (4)

Sol. $I :\sim p \rightarrow (\sim q \rightarrow \sim p)$

					(I)		(II)

p	q	~ p	~ q	$\sim q \leftrightarrow \sim$	$\sim p \rightarrow (\sim q \leftrightarrow$	$\sim q \wedge 1$	$(\sim q \wedge p) \rightarrow$
T	T	F	F	T	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	F	F	T
F	F	T	T	T	T	F	T

Both are false

Question 60. If $u = \frac{2z+i}{z+ki}$ where $z = x + iy$ and $k > 0$. Curve $Re(u) + Im(u) = 1$ cuts y-axis at two point P and Q such that PQ = 5, then value of k is

(1) 1

(2) 2

(3) 3

(4) 4

Ans. (2)

Sol. $u = \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x+(2y+1)i}{x+(y-k)i} \times \frac{x-(y-k)i}{x-(y-k)i}$

Real part of u = Re (u) = $\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of u = Im(u) = $\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now Re(u) + Im(u) = 1

$$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$$

For y-axis put x = 0 $\Rightarrow \frac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow$(2y + 1)(y – k) = (y – k)2

$\Rightarrow$ (y – k)(y + (1 + k) = 0

Y = k, -(1 + k)

Now point P(0, k), Q(0, -(1+k))

PQ = |2k + 1| = 5

2k + 1 = $\pm$5

2k = 4, -6

k = 2 , – 3

Hence k = 2 (k > 0)

Question 61. Probability of hitting a target is $\frac{1}{10}$, then find the minimum number of trials so that probability of at least one success is greater than $\frac{1}{4}$is

Ans. (3)

Sol. $p = \frac{1}{10}, q = \frac{9}{10}$

P (not hitting in n trials) $= \left(\frac{9}{10}\right)^n$

$\therefore$p (at least one hit) $= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$

$\Rightarrow \left(\frac{9}{10}\right)^2 \leq \frac{3}{4}$

$(.9)^n \leq .75$

$n = 3 \Rightarrow 0.729 \leq .75$ which is true

Question 62. Let $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, 0 < \theta < \frac{\pi}{24}$ and

$A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then which statement is false

(1) $a^2 - b^2 = \frac{1}{2}$

(2) $a^2 - b^2 \in (0,1)$

(3) $a^2 - d^2 = 0$

(4) $a^2 - c^2 = 1$

Ans. (1)

Sol. $A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 3\theta & i\sin 3\theta \\ i\sin 3\theta & \cos 3\theta \end{bmatrix}$

$A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a = \cos 5\theta, b = i\sin 5\theta, c = i\sin 5\theta, d = \cos 5\theta$

$a = d, b = c$

(1) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(2) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta \in (0,1)$ as $0 < \theta < \frac{\pi}{24}$

(3) $a^2 - d^2 = 0$

(4) $a^2 - c^2 = a^2 - b^2 = 1$

Question 63. If $\left(a - \sqrt{2}b\cos x\right)\left(a + \sqrt{2}b\cos y\right) = a^2 - b^2$ then value of $\frac{dy}{dx}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -

(1) $\frac{a-b}{a+b}$

(2) $\frac{a+b}{a-b}$

(3) $\frac{2a+b}{a-b}$

(4) $\frac{2a-b}{a-b}$

Ans. (2)

Sol.

$$\left(a-\sqrt{2}b\cos x\right)\left(-\sqrt{2}b\sin y\right)\frac{dy}{dx}+\sqrt{2}b\sin x\left(a+\sqrt{2}b\cos y\right)=0$$

$$\frac{dy}{dx}=\frac{\sqrt{2}b\sin x\left(a+\sqrt{2}b\cos y\right)}{\sqrt{2}b\sin y\left(a-\sqrt{2}b\cos x\right)}=\frac{\sin x\left(a+\sqrt{2}b\cos y\right)}{\sin y\left(a-\sqrt{2}\cos x\right)}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a+b}{a-b}$$

Question 64. $f\left(x+y\right)=f(x)+f(y)+xy^{2}+x^{2}y$ and $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$, then find value of $f'(3)$

Ans. (10)

Sol.

$$\lim_{x\to 0}\frac{f(x)}{x}=1\Rightarrow\lim_{x\to 0}\frac{f'(x)}{1}=1$$

$$\Rightarrow f'(0)=1$$

$$f(x+y)=f(x)+f(y)+xy^2+x^2y$$

$$f'(x+y)=f'(x)+0+y^2+2xy$$

Put y = -x

$$f'(0)=f'(x)+x^2-2x^2$$

$$1=f'(x)-x^2$$

$$f'(x)=1+x^2$$

$$f'(3)=10$$

Question 65. If f is twice differentiable function for $x\in R$ such that f(2) = 5, $f'(2)=8$ and $f'(x)\geq 1$, $f''(x)\geq 4$then

(1) $f(5)+f'(5)\leq 26$

(2) $f(5)+f'(5)\geq 28$

(3) $f(5)+f'(5)\leq 28$

(4) none of these

Ans. (2)

Sol. Given $f'(x)\geq 1\Rightarrow\int_2^5 f'(x)dx\geq\int_2^5 1.dx$

$$\Rightarrow \left(f(x)\right)_2^5 \geq (x)_2^5 \Rightarrow f(5)-f(2)\geq 3 \Rightarrow f(5)\geq 8 \quad \ldots (1)$$

Now $f''(x)\geq 4 \Rightarrow \int_2^5 f''(x)dx \geq \int_2^5 4dx$

$$\Rightarrow \left(f'(x)\right)_2^5 \geq (4x)_2^5$$

$$\Rightarrow f'(5)-f'(2)\geq 12$$

$$\Rightarrow f'(5)\geq 20 \quad \ldots . (2)$$

$$(1)+(2)\Rightarrow f(5)+f'(5)\geq 28$$

Question 66. If $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$, then $\frac{a_7}{a_{13}}=$

Ans. (8)

Sol. General term $\frac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1}.(3x)^{r_2}(4)^{r_3}$

$$a_7=\frac{10!.2^3.3.4^6}{3!1!6!}+\frac{10!2^2.3^3.4^5}{2!3!5!}+\frac{10!2.3^5.4^4}{1!.5!.4!}+\frac{10!.3^7.4^3}{7!3!}$$

$$a_{13}=\frac{10!2^6.3.4^3}{6!1!3!}+\frac{10!.2^5.3^3.4^2}{5!3!2!}+\frac{10!2^4 3^5.4^1}{4!5!1!}+\frac{10!.2^3.3^7.4^0}{3!7!}$$

$$\frac{a_7}{a_{13}}=2^3=8$$

Question 67. If from a point (3, 3) on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, a normal is drawn which cuts x axis at (9, 0) then value of (a2, e2) is

(1) $\left(\frac{9}{2},3\right)$

(2) $\left(\frac{9}{2},1\right)$

(3) (9, 3)

(4) (3, 9)

Ans. (1)

Sol. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

P(3, 3) lies on hyperbola then $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{9}$ .... (1)

Normal at (3, 3) is

$$\frac{a^2x}{3}+\frac{b^2y}{3}=a^2+b^2$$

Pass through (9, 0)

$$3a^2=a^2+b^2=2a^2=b^2$$

then $\frac{1}{a^2}-\frac{1}{2a^2}=\frac{1}{9}$

$$2a^2 = 9 \Rightarrow a^2 = \frac{9}{2} \text{ and } b^2 = 9$$

$$e^2 = 1 + \frac{b^2}{a^2} = 1 + 2 = 3$$